

Chapter 3

A COMPREHENSIVE REVIEW OF LACTATION CURVES IN DAIRY CATTLE

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Introduction

Livestock is an agricultural activity that has been done by mankind for centuries. People obtain animal products such as meat, milk and eggs, which have an important place in their nutrition through animal husbandry. Dairy cattle is a part of cattle breeding in livestock activities. Dairy cattle production is a field of production that is considered important by many countries because it provides milk and dairy products and fattening material [1]. Since dairy cattle breeding is a long-term activity, it is very difficult to start this activity and shift the investments made in this field to other areas. Although producers find low milk prices and high production costs, they continue their production activities for a certain period. It is very difficult for producers to abandon this activity because they make serious investments [2]. After the elimination of the problems in the yield of dairy cattle producers' expectations about the increase in the milk price increased. Price fluctuations due to the rise in milk prices and feed prices have a significant impact on the sector. Continuous monitoring of dairy establishments in the determination of the current situation and making plans for the future of the milk production sector are important in terms of increasing the efforts to ensure the sustainability of the sector [3].

Although there are important developments in dairy farming in our country in recent years, milk yield per capita and total milk production in the country is low compared to developed countries. However, milk yield has an important place in the national economy. Demand created by the increasing population in our country has made it necessary to increase animal production. Increasing animal production can be achieved either by increasing the number of animals or by

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Discussions

The mathematical function allows the lactation curve to be expressed in terms of a set of parameters to be estimated. Many mathematical models have been employed to study the lactation in dairy cattle. In this paper, a comprehensive review of more than 50 different non-linear lactation curve models used in dairy cattle has been presented by examining over 100 papers related with the mathematical functions used in modeling the lactation curves. The statistical criteria for the determination of the most appropriate model have also been given.

The most frequently used model is Wood's [13] model, which has been quoted in 92 papers. Wood's [13] model was followed by other models from [24], [26], [27], [62] [63], [64], [32], [61], [68], [35], [75] and [15]; these models were quoted 25, 7, 16, 32, 12, 16, 4, 22, 27, 12 and 10 times, respectively. As it is understood from the number of quotations of the models, the number of citations of lactation curve models proposed in recent years is few in the literature. We recommend that researchers who will work on the lactation curve choose the most and least used models when selecting a model. Because in such a study, the compatibility and incompatibility of frequently used models and less used models can be proved.

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