TURBULENCE MODELS FOR COMPUTATIONAL FLUID DYNAMICS

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PREFACE

Turbulent fluid flow is a very complicated natural phenomenon from the viewpoint of both ***understanding*** and ***analysis***. Therefore, in *turbulent motion* studies, besides the *theoretical analysis*, *statistical* and *empirical correlation* methods must also be resorted to. The most fluid flows encountered in *industry*, especially in *aeronautics*, and *civil*, *environmental, mechanical*, and *chemical engineering* are ***turbulent*** and many phenomena, such as ***heat*** or ***mass transfer***, are intimately linked to the fluid motion. In spite of the variety of ***experimental works*** *on the structure of turbulent flows* have so far been conducted, the *fundamental mechanisms* in ***turbulence phenomenon*** still remain ***incompletely clarified*** and ***many problems remain open***.

Fluid mechanics is surrounded by the difficulty that man's ability to *write* the ***governing equations of motion*** far outruns his ability to *solve them*. This difficulty is a great handicap, in the analysis of ***turbulent flows***. The ***governing equations***, can be considered to be **exact** and to apply even to the ***smallest eddies*** of turbulence. But, because ***turbulent flow*** is always *three-dimensional*, even in *one-dimensional* flow, the ***governing equations*** are *three-dimensional*, *unsteady*, *nonlinear* ***partial differential equations***.

Turbulence is a *natural phenomenon* and the investigation of it relies on ***traditional basic concepts***. There are two ways to study the *fluid turbulence*: **exact science method** and **model method**. The method of ***exact science*** is based on *fundamental laws and principles* of ***physics*** by applying ***mathematics*** and *supported* by ***experimental work***. On the other hand, the ***modeling approach*** is *heavily based on* ***empiricism***.

A **turbulence model** is a composition of ***model equations***. They describe the turbulent flow phenomenon, which cannot perhaps be the actual turbulence, but adequately close to it for representing a ***useful*** and ***simplified*** nature of the happening. The *accuracy* of the ***model*** and *its capability* to represent the characteristics of a turbulent flow, are directly dependent on the existing knowledge of the *physics of the phenomenon* that it has been possible to formulate in the ***model equations***.

Actually, the **turbulence model** is a *qualitative* and *quantitative* ***summary*** of the present knowledge ***on fluid turbulence***. Accordingly, the ***mathematical models*** related to ***turbulent flow*** are open to continuous improvement, enriched by ***new concepts*** inspired by ***experimental findings*** or ***numerical simulations*** or progress achieved in ***theoretical approaches***.

Currently, the subject of ***real-fluid flow analysis*** lies in an *awkward transition* from the ***traditional mathematical approach*** toward ***digital-computer simulations***. Traditional ***mathematical boundary-flow analysis***, *supported* by ***experimental data***, gives *good insight* into ***viscous flow*** but limited to *certain approximations* and *simple geometries* only. ***Computer modeling***, on the other hand, is also successfully applicable to ***nonboundary-layer problems*** but gives less insight and is restricted by ***grid-storage*** and ***truncation-accuracy*** limitations. Moreover, the ***computational turbulence modeling*** has distinct *physical* and *geometric* limitations.

The advent of ***supercomputers*** in recent years, has led the ***investigators*** and the ***practicing engineers*** to discard detailed study of ***analytical methods*** because ***every purely theoretical approach*** does not lead to *practical prediction methods*. Therefore, among the existing ***theories*** and ***models*** for the turbulent flow studies, preference has been given to the methods of ***numerical predictions***. So, the ***numerical prediction*** *of turbulent flows* is now becoming *more and more important* for ***practical applications***. In fact, the ***exact science method*** and the ***model method***, for the analysis of turbulent flow, are to some extent *complementary*.

The *aim of this book* is to present the ***basic theory of turbulent flow*** and the methods of ***turbulence closure modeling*** for ***Computational Fluid Dynamics*** (**CFD**). The *last chapter* of the book is devoted to the *summary of results* of the investigations that have been carried out in ***Civil Engineering Department of Çukurova University***, concerning the **CFD** ***simulation of turbulent flow*** for some ***engineering applications***.

The material presented in this book is appropriate as a ***reference text*** for a ***postgraduate*** **CFD** ***course*** and for ***practicing engineers***. We hope that this book will be beneficial for the *scientists*, *students* and *practicing engineers* in their efforts of **CFD** ***simulation of turbulent flow***.

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NOTATION

A	Surface area
a_i	Acceleration in tensor notation
a_{ij}, b_{ij}	Reynolds-stress anisotropy tensor
Cn	Courant number
C_μ	Model constant
C_{ij}	Cross-term stresses
c	Concentration of matter
c_f	Local skin friction
D	Diffusion coefficient
D_t	Turbulent mass diffusivity
DNS	Direct numerical simulation
E	Stored energy
$E(\kappa)$, $E(f)$	Turbulence kinetic energy of each wave
F	Volume fraction
F_{kleb}	Klebanoff's intermittency function
F_{ij}	System rotation term
F_1, F_2	Blending function
Fr	Froude number
FVM	Finite volume method
f	Frequency, Damping function
f_i	Mass diffusion flux, Body force per unit mass
f_μ	Damping function for C_μ
G	Shape factor
Gr	Grashof number
G_{ij}	Buoyant production term
GCI	Grid convergence index
g	Gravitational acceleration
H(x)	Ramp function
h	Flow depth
I	Turbulence intensity
K	Mean kinetic energy
k	Turbulence kinetic energy
k_l	Laminar kinetic energy
k_s	Nikuradse's equivalent sand roughness

k_{sgs}	SGS kinetic energy
L	Characteristic length scale
L_{int}	Integral length scale
L_{ij}	Leonard stresses
LES	Large eddy simulation
ℓ	Turbulence length scale
ℓ_m	Mixing-length
ℓ_o	Large eddy length scale
ℓ_η	Small eddy length scale
ℓ_μ, ℓ_ε	Turbulence length scales
Le	Lewis number
MSE	Mean square error
P_k	TKE production term
Pr	Prandtl number
p	Instantaneous pressure, Order of accuracy
$\overline{p}$	Mean pressure
p_e	Pressure at the outer edge of boundary layer
p'	Pressure fluctuation
Q	Heat, Flow rate
R	Radius of pipe, Averaged effect of breakdown of fluctuations
Ra	Rayleigh number
Ri	Richardson number
R_{ij}	Correlation coefficient, Reynolds stress tensor
R_{nat}	Natural transition production term
Re	Reynolds number
Re_ℓ	Local turbulent Reynolds number
Re_t	Turbulent Reynolds number
RANS	Reynolds-averaged Navier-Stokes equation
r	Grid refinement factor
Sc	Schmidt number
S_r	Tuning parameter
S_{ij}	Mean strain-rate tensor, Mean angular deformation rate tensor
SGS	Subgrid-scale
s_{ij}	Fluctuating strain-rate tensor
T	Duration, Period of eddies
T_m	Time scale of molecular diffusion
T_t	Characteristic time scale
TKE	Turbulence kinetic energy

t	Time
$\hat{U}$	Internal energy
$\hat{u}$	Internal energy per unit mass
u, v, w	Instantaneous velocity components
$\bar{u}, \bar{v}, \bar{w}$	Mean velocity components
u', v', w'	Velocity fluctuations
u_i	Instantaneous velocity in tensor notation
$\bar{u}_i$	Mean velocity in tensor notation
$\bar{u}_m$	Freestream mean velocity (shear-layer edge velocity)
u'_i	Velocity fluctuation in tensor notation
u_*	Friction (shear) velocity
u'_{rms}	Root mean square of the fluctuation velocity
u^+	Dimensionless velocity
x, y, z	Cartesian coordinates
x_i	Position in tensor notation
V	Instantaneous velocity
$\bar{V}$	Mean velocity
V'	Velocity fluctuation
VOF	Volume of fluid
$\forall$	Volume
W	Work
w	Wake function
Y	Yap length scale correction
y^+	Dimensionless wall distance
α	Thermal diffusivity
β	Clauser's equilibrium parameter
Γ	Generalized diffusion coefficient
γ	Intermittency factor, Unit weight of fluid
δ	Boundary layer thickness
δ^*	Displacement thickness
Δ	Defect thickness
Δt	Time step
ε	Turbulence kinetic energy dissipation rate
ζ_{ij}	Vorticity tensor
θ	Temperature
κ	Wave number, Karman constant

λ	Spatial Taylor microscale
λ_ε	Blending function
μ	Dynamic (molecular) viscosity
μ_{eff}	Effective viscosity
μ_t	Eddy (turbulent) viscosity
ν	Kinematic (molecular) viscosity
ν_t	Kinematic eddy (turbulent) viscosity
Π	Profile parameter
Π_{ij}	Pressure-strain correlation term
ρ	Fluid density
$\overline{\rho}$	Mean fluid density
ρ'	Fluid density fluctuation
σ	Standard deviation, Prandtl-Schmidt number
σ_k	Turbulent Prandtl number for k
σ_ε	Turbulent Prandtl number for ε
σ_ω	Turbulent Prandtl number for ω
σ_x, σ_y, σ_z	Normal stresses in x-, y- and z-direction
τ	Turbulence time scale, Shear stress
τ_o	Turbulence time large-scale, Boundary shear stress
τ_η	Turbulence time small-scale
τ_m	Maximum Reynolds shear stress
τ_{ij}	Stress tensor, SGS stress tensor
τ'_{ij}	Reynolds stresses
υ	Characteristic velocity (turbulence velocity scale)
υ_o	Turbulence velocity large-scale
υ_η	Turbulence velocity small-scale
Ω_{ij}	Rotation rate tensor, Vorticity tensor
ϕ	Arbitrary scalar quantity per unit mass
ϕ'	Fluctuation of arbitrary scalar quantity per unit mass
ω	Specific dissipation rate of TKE

CONTENTS

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INDEX