

TURBULENCE MODELS FOR COMPUTATIONAL FLUID DYNAMICS

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PREFACE

Turbulent fluid flow is a very complicated natural phenomenon from the viewpoint of both *understanding* and *analysis*. Therefore, in *turbulent motion* studies, besides the *theoretical analysis*, *statistical* and *empirical correlation* methods must also be resorted to. The most fluid flows encountered in *industry*, especially in *aeronautics*, and *civil, environmental, mechanical*, and *chemical engineering* are *turbulent* and many phenomena, such as *heat* or *mass transfer*, are intimately linked to the fluid motion. In spite of the variety of *experimental works* on the structure of *turbulent flows* have so far been conducted, the *fundamental mechanisms* in *turbulence phenomenon* still remain *incompletely clarified* and *many problems remain open*.

Fluid mechanics is surrounded by the difficulty that man's ability to write the *governing equations of motion* far outruns his ability to *solve them*. This difficulty is a great handicap, in the analysis of *turbulent flows*. The *governing equations*, can be considered to be *exact* and to apply even to the *smallest eddies* of turbulence. But, because *turbulent flow* is always *three-dimensional*, even in *one-dimensional* flow, the *governing equations* are *three-dimensional*, *unsteady*, *nonlinear partial differential equations*.

Turbulence is a *natural phenomenon* and the investigation of it relies on *traditional basic concepts*. There are two ways to study the *fluid turbulence*: *exact science method* and *model method*. The method of *exact science* is based on *fundamental laws and principles* of *physics* by applying *mathematics* and supported by *experimental work*. On the other hand, the *modeling approach* is heavily based on *empiricism*.

A *turbulence model* is a composition of *model equations*. They describe the turbulent flow phenomenon, which cannot perhaps be the actual turbulence, but adequately close to it for representing a *useful* and *simplified* nature of the happening. The *accuracy* of the *model* and *its capability* to represent the characteristics of a turbulent flow, are directly dependent on the existing knowledge of the *physics of the phenomenon* that it has been possible to formulate in the *model equations*.

Actually, the *turbulence model* is a *qualitative* and *quantitative summary* of the present knowledge *on fluid turbulence*. Accordingly, the *mathematical models* related to *turbulent flow* are open to continuous improvement,

enriched by *new concepts* inspired by *experimental findings* or *numerical simulations* or progress achieved in *theoretical approaches*.

Currently, the subject of *real-fluid flow analysis* lies in an *awkward transition* from the *traditional mathematical approach* toward *digital-computer simulations*. Traditional *mathematical boundary-flow analysis*, supported by *experimental data*, gives good insight into *viscous flow* but limited to *certain approximations* and *simple geometries* only. *Computer modeling*, on the other hand, is also successfully applicable to *nonboundary-layer problems* but gives less insight and is restricted by *grid-storage* and *truncation-accuracy* limitations. Moreover, the *computational turbulence modeling* has distinct *physical* and *geometric* limitations.

The advent of *supercomputers* in recent years, has led the *investigators* and the *practicing engineers* to discard detailed study of *analytical methods* because *every purely theoretical approach* does not lead to *practical prediction methods*. Therefore, among the existing *theories* and *models* for the turbulent flow studies, preference has been given to the methods of *numerical predictions*. So, the *numerical prediction* of turbulent flows is now becoming *more and more important* for *practical applications*. In fact, the *exact science method* and the *model method*, for the analysis of turbulent flow, are to some extent *complementary*.

The aim of this book is to present the *basic theory of turbulent flow* and the methods of *turbulence closure modeling* for *Computational Fluid Dynamics* (CFD). The last chapter of the book is devoted to the *summary of results* of the investigations that have been carried out in *Civil Engineering Department of Çukurova University*, concerning the *CFD simulation of turbulent flow* for some *engineering applications*.

The material presented in this book is appropriate as a *reference text* for a *postgraduate CFD course* and for *practicing engineers*. We hope that this book will be beneficial for the *scientists*, *students* and *practicing engineers* in their efforts of *CFD simulation of turbulent flow*.

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NOTATION

A	Surface area
a_i	Acceleration in tensor notation
a_{ij}, b_{ij}	Reynolds-stress anisotropy tensor
C_n	Courant number
C_μ	Model constant
C_{ij}	Cross-term stresses
c	Concentration of matter
c_f	Local skin friction
D	Diffusion coefficient
D_t	Turbulent mass diffusivity
DNS	Direct numerical simulation
E	Stored energy
$E(\kappa), E(f)$	Turbulence kinetic energy of each wave
F	Volume fraction
F_{kleb}	Klebanoff's intermittency function
F_{ij}	System rotation term
F_1, F_2	Blending function
Fr	Froude number
FVM	Finite volume method
f	Frequency, Damping function
f_i	Mass diffusion flux, Body force per unit mass
f_μ	Damping function for C_μ
G	Shape factor
Gr	Grashof number
G_{ij}	Buoyant production term
GCI	Grid convergence index
g	Gravitational acceleration
$H(x)$	Ramp function
h	Flow depth
I	Turbulence intensity
K	Mean kinetic energy
k	Turbulence kinetic energy
k_l	Laminar kinetic energy
k_s	Nikuradse's equivalent sand roughness

k_{sgs}	SGS kinetic energy
L	Characteristic length scale
L_{int}	Integral length scale
L_{ij}	Leonard stresses
LES	Large eddy simulation
ℓ	Turbulence length scale
ℓ_m	Mixing-length
ℓ_o	Large eddy length scale
ℓ_η	Small eddy length scale
$\ell_\mu, \ell_\varepsilon$	Turbulence length scales
Le	Lewis number
MSE	Mean square error
P_k	TKE production term
Pr	Prandtl number
p	Instantaneous pressure, Order of accuracy
\bar{p}	Mean pressure
p_e	Pressure at the outer edge of boundary layer
p'	Pressure fluctuation
Q	Heat, Flow rate
R	Radius of pipe, Averaged effect of breakdown of fluctuations
Ra	Rayleigh number
Ri	Richardson number
R_{ij}	Correlation coefficient, Reynolds stress tensor
R_{nat}	Natural transition production term
Re	Reynolds number
Re_ℓ	Local turbulent Reynolds number
Re_t	Turbulent Reynolds number
RANS	Reynolds-averaged Navier-Stokes equation
r	Grid refinement factor
Sc	Schmidt number
S_r	Tuning parameter
S_{ij}	Mean strain-rate tensor, Mean angular deformation rate tensor
SGS	Subgrid-scale
s_{ij}	Fluctuating strain-rate tensor
T	Duration, Period of eddies
T_m	Time scale of molecular diffusion
T_t	Characteristic time scale
TKE	Turbulence kinetic energy

t	Time
\hat{U}	Internal energy
\hat{u}	Internal energy per unit mass
u, v, w	Instantaneous velocity components
$\bar{u}, \bar{v}, \bar{w}$	Mean velocity components
u', v', w'	Velocity fluctuations
u_i	Instantaneous velocity in tensor notation
\bar{u}_i	Mean velocity in tensor notation
\bar{u}_m	Freestream mean velocity (shear-layer edge velocity)
u'_i	Velocity fluctuation in tensor notation
u_*	Friction (shear) velocity
u'_{rms}	Root mean square of the fluctuation velocity
u^+	Dimensionless velocity
x, y, z	Cartesian coordinates
x_i	Position in tensor notation
V	Instantaneous velocity
\bar{V}	Mean velocity
V'	Velocity fluctuation
VOF	Volume of fluid
\forall	Volume
W	Work
w	Wake function
Y	Yap length scale correction
y^+	Dimensionless wall distance
α	Thermal diffusivity
β	Clauser's equilibrium parameter
Γ	Generalized diffusion coefficient
γ	Intermittency factor, Unit weight of fluid
δ	Boundary layer thickness
δ^*	Displacement thickness
Δ	Defect thickness
Δt	Time step
ε	Turbulence kinetic energy dissipation rate
ζ_{ij}	Vorticity tensor
θ	Temperature
κ	Wave number, Karman constant

VIII

λ	Spatial Taylor microscale
λ_ε	Blending function
μ	Dynamic (molecular) viscosity
μ_{eff}	Effective viscosity
μ_t	Eddy (turbulent) viscosity
ν	Kinematic (molecular) viscosity
ν_t	Kinematic eddy (turbulent) viscosity
Π	Profile parameter
Π_{ij}	Pressure-strain correlation term
ρ	Fluid density
$\bar{\rho}$	Mean fluid density
ρ'	Fluid density fluctuation
σ	Standard deviation, Prandtl-Schmidt number
σ_k	Turbulent Prandtl number for k
σ_ε	Turbulent Prandtl number for ε
σ_ω	Turbulent Prandtl number for ω
$\sigma_x, \sigma_y, \sigma_z$	Normal stresses in x-, y- and z-direction
τ	Turbulence time scale, Shear stress
τ_o	Turbulence time large-scale, Boundary shear stress
τ_η	Turbulence time small-scale
τ_m	Maximum Reynolds shear stress
τ_{ij}	Stress tensor, SGS stress tensor
τ'_{ij}	Reynolds stresses
u	Characteristic velocity (turbulence velocity scale)
u_o	Turbulence velocity large-scale
u_η	Turbulence velocity small-scale
Ω_{ij}	Rotation rate tensor, Vorticity tensor
ϕ	Arbitrary scalar quantity per unit mass
ϕ'	Fluctuation of arbitrary scalar quantity per unit mass
ω	Specific dissipation rate of TKE

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