TURBULENCE MODELS FOR COMPUTATIONAL FLUID DYNAMICS

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PREFACE

Turbulent fluid flow is a very complicated natural phenomenon from the viewpoint of both ***understanding*** and ***analysis***. Therefore, in *turbulent motion* studies, besides the *theoretical analysis*, *statistical* and *empirical correlation* methods must also be resorted to. The most fluid flows encountered in *industry*, especially in *aeronautics*, and *civil*, *environmental, mechanical*, and *chemical engineering* are ***turbulent*** and many phenomena, such as ***heat*** or ***mass transfer***, are intimately linked to the fluid motion. In spite of the variety of ***experimental works*** *on the structure of turbulent flows* have so far been conducted, the *fundamental mechanisms* in ***turbulence phenomenon*** still remain ***incompletely clarified*** and ***many problems remain open***.

Fluid mechanics is surrounded by the difficulty that man's ability to *write* the ***governing equations of motion*** far outruns his ability to *solve them*. This difficulty is a great handicap, in the analysis of ***turbulent flows***. The ***governing equations***, can be considered to be **exact** and to apply even to the ***smallest eddies*** of turbulence. But, because ***turbulent flow*** is always *three-dimensional*, even in *one-dimensional* flow, the ***governing equations*** are *three-dimensional*, *unsteady*, *nonlinear* ***partial differential equations***.

Turbulence is a *natural phenomenon* and the investigation of it relies on ***traditional basic concepts***. There are two ways to study the *fluid turbulence*: **exact science method** and **model method**. The method of ***exact science*** is based on *fundamental laws and principles* of ***physics*** by applying ***mathematics*** and *supported* by ***experimental work***. On the other hand, the ***modeling approach*** is *heavily based on* ***empiricism***.

A **turbulence model** is a composition of ***model equations***. They describe the turbulent flow phenomenon, which cannot perhaps be the actual turbulence, but adequately close to it for representing a ***useful*** and ***simplified*** nature of the happening. The *accuracy* of the ***model*** and *its capability* to represent the characteristics of a turbulent flow, are directly dependent on the existing knowledge of the *physics of the phenomenon* that it has been possible to formulate in the ***model equations***.

Actually, the **turbulence model** is a *qualitative* and *quantitative* ***summary*** of the present knowledge ***on fluid turbulence***. Accordingly, the ***mathematical models*** related to ***turbulent flow*** are open to continuous improvement, enriched by ***new concepts*** inspired by ***experimental findings*** or ***numerical simulations*** or progress achieved in ***theoretical approaches***.

Currently, the subject of ***real-fluid flow analysis*** lies in an *awkward transition* from the ***traditional mathematical approach*** toward ***digital-computer simulations***. Traditional ***mathematical boundary-flow analysis***, *supported* by ***experimental data***, gives *good insight* into ***viscous flow*** but limited to *certain approximations* and *simple geometries* only. ***Computer modeling***, on the other hand, is also successfully applicable to ***nonboundary-layer problems*** but gives less insight and is restricted by ***grid-storage*** and ***truncation-accuracy*** limitations. Moreover, the ***computational turbulence modeling*** has distinct *physical* and *geometric* limitations.

The advent of ***supercomputers*** in recent years, has led the ***investigators*** and the ***practicing engineers*** to discard detailed study of ***analytical methods*** because ***every purely theoretical approach*** does not lead to *practical prediction methods*. Therefore, among the existing ***theories*** and ***models*** for the turbulent flow studies, preference has been given to the methods of ***numerical predictions***. So, the ***numerical prediction*** *of turbulent flows* is now becoming *more and more important* for ***practical applications***. In fact, the ***exact science method*** and the ***model method***, for the analysis of turbulent flow, are to some extent *complementary*.

The *aim of this book* is to present the ***basic theory of turbulent flow*** and the methods of ***turbulence closure modeling*** for ***Computational Fluid Dynamics*** (**CFD**). The *last chapter* of the book is devoted to the *summary of results* of the investigations that have been carried out in ***Civil Engineering Department of Çukurova University***, concerning the **CFD** ***simulation of turbulent flow*** for some ***engineering applications***.

The material presented in this book is appropriate as a ***reference text*** for a ***postgraduate*** **CFD** ***course*** and for ***practicing engineers***. We hope that this book will be beneficial for the *scientists*, *students* and *practicing engineers* in their efforts of **CFD** ***simulation of turbulent flow***.

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NOTATION

A	Surface area
a_i	Acceleration in tensor notation
a_{ij}, b_{ij}	Reynolds-stress anisotropy tensor
Cn	Courant number
C_μ	Model constant
C_{ij}	Cross-term stresses
c	Concentration of matter
c_f	Local skin friction
D	Diffusion coefficient
D_t	Turbulent mass diffusivity
DNS	Direct numerical simulation
E	Stored energy
$E(\kappa)$, $E(f)$	Turbulence kinetic energy of each wave
F	Volume fraction
F_{kleb}	Klebanoff's intermittency function
F_{ij}	System rotation term
F_1, F_2	Blending function
Fr	Froude number
FVM	Finite volume method
f	Frequency, Damping function
f_i	Mass diffusion flux, Body force per unit mass
f_μ	Damping function for C_μ
G	Shape factor
Gr	Grashof number
G_{ij}	Buoyant production term
GCI	Grid convergence index
g	Gravitational acceleration
H(x)	Ramp function
h	Flow depth
I	Turbulence intensity
K	Mean kinetic energy
k	Turbulence kinetic energy
k_l	Laminar kinetic energy
k_s	Nikuradse's equivalent sand roughness

k_{sgs}	SGS kinetic energy
L	Characteristic length scale
L_{int}	Integral length scale
L_{ij}	Leonard stresses
LES	Large eddy simulation
ℓ	Turbulence length scale
ℓ_m	Mixing-length
ℓ_o	Large eddy length scale
ℓ_η	Small eddy length scale
ℓ_μ, ℓ_ε	Turbulence length scales
Le	Lewis number
MSE	Mean square error
P_k	TKE production term
Pr	Prandtl number
p	Instantaneous pressure, Order of accuracy
$\overline{p}$	Mean pressure
p_e	Pressure at the outer edge of boundary layer
p'	Pressure fluctuation
Q	Heat, Flow rate
R	Radius of pipe, Averaged effect of breakdown of fluctuations
Ra	Rayleigh number
Ri	Richardson number
R_{ij}	Correlation coefficient, Reynolds stress tensor
R_{nat}	Natural transition production term
Re	Reynolds number
Re_ℓ	Local turbulent Reynolds number
Re_t	Turbulent Reynolds number
RANS	Reynolds-averaged Navier-Stokes equation
r	Grid refinement factor
Sc	Schmidt number
S_r	Tuning parameter
S_{ij}	Mean strain-rate tensor, Mean angular deformation rate tensor
SGS	Subgrid-scale
s_{ij}	Fluctuating strain-rate tensor
T	Duration, Period of eddies
T_m	Time scale of molecular diffusion
T_t	Characteristic time scale
TKE	Turbulence kinetic energy

t	Time
$\hat{U}$	Internal energy
$\hat{u}$	Internal energy per unit mass
u, v, w	Instantaneous velocity components
$\bar{u}, \bar{v}, \bar{w}$	Mean velocity components
u', v', w'	Velocity fluctuations
u_i	Instantaneous velocity in tensor notation
$\bar{u}_i$	Mean velocity in tensor notation
$\bar{u}_m$	Freestream mean velocity (shear-layer edge velocity)
u'_i	Velocity fluctuation in tensor notation
u_*	Friction (shear) velocity
u'_{rms}	Root mean square of the fluctuation velocity
u^+	Dimensionless velocity
x, y, z	Cartesian coordinates
x_i	Position in tensor notation
V	Instantaneous velocity
$\bar{V}$	Mean velocity
V'	Velocity fluctuation
VOF	Volume of fluid
$\forall$	Volume
W	Work
w	Wake function
Y	Yap length scale correction
y^+	Dimensionless wall distance
α	Thermal diffusivity
β	Clauser's equilibrium parameter
Γ	Generalized diffusion coefficient
γ	Intermittency factor, Unit weight of fluid
δ	Boundary layer thickness
δ^*	Displacement thickness
Δ	Defect thickness
Δt	Time step
ε	Turbulence kinetic energy dissipation rate
ζ_{ij}	Vorticity tensor
θ	Temperature
κ	Wave number, Karman constant

λ	Spatial Taylor microscale
λ_ε	Blending function
μ	Dynamic (molecular) viscosity
μ_{eff}	Effective viscosity
μ_t	Eddy (turbulent) viscosity
ν	Kinematic (molecular) viscosity
ν_t	Kinematic eddy (turbulent) viscosity
Π	Profile parameter
Π_{ij}	Pressure-strain correlation term
ρ	Fluid density
$\bar{\rho}$	Mean fluid density
ρ'	Fluid density fluctuation
σ	Standard deviation, Prandtl-Schmidt number
σ_k	Turbulent Prandtl number for k
σ_ε	Turbulent Prandtl number for ε
σ_ω	Turbulent Prandtl number for ω
σ_x, σ_y, σ_z	Normal stresses in x-, y- and z-direction
τ	Turbulence time scale, Shear stress
τ_o	Turbulence time large-scale, Boundary shear stress
τ_η	Turbulence time small-scale
τ_m	Maximum Reynolds shear stress
τ_{ij}	Stress tensor, SGS stress tensor
τ'_{ij}	Reynolds stresses
υ	Characteristic velocity (turbulence velocity scale)
υ_o	Turbulence velocity large-scale
υ_η	Turbulence velocity small-scale
Ω_{ij}	Rotation rate tensor, Vorticity tensor
ϕ	Arbitrary scalar quantity per unit mass
ϕ'	Fluctuation of arbitrary scalar quantity per unit mass
ω	Specific dissipation rate of TKE

CONTENTS

Preface III
Notation V

1 Preliminary Concepts of Fluid Flow 1

1.1 Introduction 1
1.1.1 Description of '*Computational Fluid Dynamics*' 1
1.1.2 CFD Code Structure 2
1.2 Description of Flow Field 5
1.3 State of Stress at a Point 5
1.4 Convective Motion of Fluid: Strain-Rate, Rotation-Rate, and Vorticity 8
1.5 Stress-Strain Rate Relationships: Constitutive Equations 14
1.6 Transport Properties of a Fluid 15
1.7 Mass Transfer – Coefficient of Mass Diffusivity 16
1.8 Momentum Transfer – Coefficient of Viscosity 17
1.9 Heat Transfer – Coefficient of Thermal Conductivity 18

2 Governing Equations of Laminar Flow 21

2.1 Conservation Laws: Transport Equations 21
2.1.1 Conservation of Mass: *CONTINUITY EQUATION* 22
2.1.2 Conservation of Momentum: *EQUATION OF MOTION* 25
2.1.3 Conservation of Energy 36
2.2 Classification of Physical Behavior of Fluid Flow 42
2.2.1 Equilibrium Problems 43
2.2.2 Marching Problems 43
2.3 Partial Differential Equations for Fluid Flow 45
2.3.1 Classification Method for Partial Differential Equations (PDEs) 45
2.3.2 Classification of Fluid Flow Equations 46
2.3.3 Initial and Boundary Conditions for Viscous Fluid Flow Equations 47

3 Turbulence in Fluid Flow 49

3.1 Introduction 49
3.2 General Properties of Turbulent Flow 50
3.2.1 Characteristic Features of Turbulence 50
3.2.2 Random Nature of Turbulence 53

3.2.3 Diffusivity of Turbulence 54
3.2.4 Length Scales in Turbulent Flows 56
3.2.5 Methods of Analysis of Turbulent Flow 59

3.3 Statistical Description of Turbulence 60

3.3.1 Probability Density Function 60
3.3.2 Mean Product of Velocity Fluctuations 67
3.3.3 Analysis of Correlations of Turbulence 72
3.3.4 Turbulence Scales and *RICHARDSON*'s *ENERGY CASCADE* 85
3.3.5 Turbulence Energy Spectrum 88
3.3.6 Cascade Process Created by Vorticity: *Vortex Stretching and Vortex Tilting* 103

3.4 Velocity Profiles: Viscous Inner, Logarithmic Overlap and Outer Layers of Boundary Flows 106

3.5 Some Typical Turbulent Flows 113

3.5.1 Complex Shear Driven Turbulent Flows 113
3.5.2 Buoyancy-Driven Turbulent Flows 114

4 Governing Equations of Turbulent Flow 117

4.1 Introduction 117

4.1.1 Time Averaging of a Dependent Variable in Turbulent Flow 117
4.1.2 Stress, Strain-Rate and Energy in Turbulent Flow 118

4.2 Conservation of Mass: *CONTINUITY EQUATION* 119

4.3 Conservation of Momentum: *EQUATIONS OF MOTION* – RANS 120

4.3.1 Momentum Transport Equations for RANS 126

4.4 General Transport Equation for Turbulent Flow 127

4.4.1 Turbulent Transport of Scalar Quantities 128
4.4.2 Linear Constitutive Relation – *Boussinesq Approximation* 133
4.4.3 Mixing-Length Hypotheses 135

4.5 Vorticity Equation for Turbulent Flow 142

4.6 Conservation of Energy for Turbulent Flow 146

4.6.1 Kinetic Energy of Mean Flow 146
4.6.2 Time-Averaged Total Kinetic Energy 147
4.6.3 Turbulence Kinetic Energy 149
4.6.4 Rearrangement of Kinetic Energy of Mean Flow 151
4.6.5 Rearrangement of Turbulence Kinetic Energy 154
4.6.6 Integration of Eqs. 4.66 and 4.68 155

4.7 Transport Equations for Kinetic Energy 157

4.7.1 Exact k-Transport Equation 157
4.7.2 Modeled k-Transport Equation 165
4.7.3 Exact and Modeled ε-Transport Equation 168
4.7.4 Exact K-Transport Equation 169

4.8 Exact Transport Equations for Reynolds Stresses 172

5 Turbulence Modeling 181

5.1 Introduction 181

5.2 Turbulence Closure Models 183

5.2.1 RANS-Based Turbulence Models (*Classical Models*) 185
5.2.1-1 Linear Eddy Viscosity Models (LEVM) 186
5.2.1-1.1 Zero-Equation Models 186
5.2.1-1.2 One-Equation Models 195
5.2.1-1.3 Two-Equation Models 201
5.2.1-1.3.1 The k-ε Models 209
5.2.1-1.3.2 The k-τ Model 242
5.2.1-1.3.3 The k-ω Models 243
5.2.1-1.3.4 The Menter k-ω Models 250
5.2.1-1.3.5 The Transition k-ω Models 257
5.2.1-2 Nonlinear Eddy Viscosity Models (NEVM) 263
5.2.1-2.1 Nonlinear Constitutive Relation Models 263
5.2.1-2.1.1 Nonlinear k-ε Models 264
5.2.1-2.1.1.1 Quadratic Stress-Strain Relation Models 265
5.2.1-2.1.1.2 Cubic Stress-Strain Relation Models 272
5.2.1-2.1.2 Algebraic Stress Models (ASM) 273
5.2.1-2.1.2.1. Implicit Algebraic Stress Models (IASM) 274
5.2.1-2.1.2.2 Explicit Algebraic Stress Models (EASM) 276
5.2.1-2.2 The v'^2-f Models 281
5.2.1-3 Reynolds Stress Models (RSM) 285
5.2.2 Direct Numerical Simulation (DNS) 309
5.2.3 Large Eddy Simulation (LES) 313
5.2.3-1 Subgrid-Scale Models (SGS) 321
5.2.4 Detached Eddy Simulation (DES) 329

6 Errors and Uncertainties in CFD Modeling: *VERIFICATION and VALIDATION* 335

6.1 Errors and Uncertainties 335

6.1.1 Numerical Errors 336
6.1.2 Input Uncertainties 341
6.1.3 Physical Model Uncertainties 343

6.2 Verification and Validation 344

6.2.1 Verification Process of CFD Modeling 344
6.2.2 Validation Process of CFD Modeling 349

6.3 Discretization of Solution Domain: *GRID DESIGN* 351

6.3.1 Introduction 351
6.3.2 Grid Types: Control Volume Choices 352
6.3.3 Mesh Quality 356

6.4 Reporting the Inputs and Results of CFD Simulations 359

7 Engineering Applications of CFD Simulation 363

7.1 Basic Information for the Applications 363

7.1.1 Volume of Fluid Method for Free-Surface Modeling 363
7.1.2 Numerical Solution 366

7.2 Engineering Applications 367

7.2.1 Experimental and Numerical Modeling of Flow Over a Broad-Crested Weir 367
7.2.2 Experimental and Numerical Modeling of Flow Over a Gate-Controlled Broad-Crested Weir 372
7.2.3 Numerical Modeling of B-Type Hydraulic Jump at an Abrupt Drop 376
7.2.4 Numerical Modeling of Submerged Hydraulic Jump From a Sluice Gate 385
7.2.5 Numerical Simulation of Flow Over a Semicylinder Weir 393
7.2.6 Experimental and Numerical Modeling of Flow over a Gate-Controlled Semicylinder Weir 403
7.2.7 Numerical Simulation of Flow Over a Curvilinear Broad-Crested Weir 408
7.2.8 Numerical Modeling of Submerged Hydraulic Jump Downstream of a Drum Gate 417
7.2.9 Experimental and Numerical Modeling of Flow Over a Spillway 422
7.2.10 Numerical Modeling of Trapezoidal Weir Flow with RANS-Based, LES and DES Models 427

References 435
Index 445

REFERENCES

Absi, R. (2006). *A roughness and time dependent mixing length equation*. Journal of Hydraulic, Coastal and Environmental Eng., Japan Society of Civil Engineers, 62(4), 437-446.

AIAA (1998). *Guide for the Verification and Validation of Computational Fluid Dynamics Simulations*. American Institute of Aeronautics and Astronautics Guide G-077-1998.

Aköz, M. S., Gümüş, V. and Kırkgöz, M.S. (2014). *Numerical simulation of flow over a semicylinder weir*. Journal of Irrigation and Drainage Engineering, ASCE, ISSN 0733-9437/04014016(11).

Ansys Inc. (2011). *ANSYS FLUENT Theory Guide*, Release 14.0.

Ashgriz, N., Barbat, T. and Wang, G. (2004). *A computational Lagrangian-Eulerian advection remap for free surface flows*. Int. Journal for Numerical Methods in Fluids, 44(1), 1-32.

Bal, H., Kırkgöz, M. S. and Gümüş, V. (2011). *Experimental and numerical modeling of flow over a broad-crested weir*. Çukurova University Journal of the Faculty of Engineering and Architecture, 26(2), 33-44.

Balaras, E. and Benocci, C. (1994). *Subgrid-scale models in finite-difference simulations of complex wall bounded flows*. AGARD CP 551, pp. 2.1-2.5, Neuilly-Sur-Seine, France: AGARD.

Baldwin, B. S. and Barth, T. J. (1990). *Thin-layer approximation and algebraic model for separated turbulent flow*. AIAA Paper 78-257.

Baldwin, B. S. and Lomax, H. (1978). *A One-Equation Turbulence Transport Model for High Reynolds Number Wall-Bounded Flows*. NASA TM-102847.

Boussinesq, J. (1877). *Théorie de l'écoulement tourbillant*. Mém. Prés. Acad. XXIII, 46, Paris.

Bradshaw, P. (1973). *Effect of Streamline Curvature on Turbulent Flow*. Agardograph Progress No. 169, AGARD.

Cebeci, T. and Smith, A. M. O. (1974). *Analysis of Turbulent Boundary Layers*. Academic Press, Inc., New York.

Celik, I. B., Ghia, U., Roache, P. J., Freitas, C. J., Coleman H. and Raad, P. R. (2008). *Procedure for estimation and reporting of uncertainty due to discretization in* CFD *applications*. J. Fluids Engineering, ASME, 130-078001, 1-4.

Chau, P. Y. (1945). *On the velocity correlations and the solution of the equations of turbulent fluctuation*. Quart. Appl. Math., 3, 38-54.

Chen, H. C. and Patel, V. C. (1988). *Near wall turbulence models for complex flows including separation*. AIAA Journal., 26(6), 641-656.

Chien, K. (1982). *Predictions of channel and boundary layer flows with a low Reynolds number turbulence model*. AIAA Journal, 20, 33-38.

Clauser, F. H. (1954). *Turbulent boundary layers in adverse pressure gradients*. J. Aeronautical Sciences, 21(2), 91-108.

Clauser, F. H. (1956). *The Turbulent Boundary Layer*. Advances in Applied Mechanics, 4, 1-51, Academic Press, New York, NY.

Coleman, H. W. and Stern, F. (1997). *Uncertainties and CFD code validation*. J. Fluids Eng., Trans. ASME, 199, 795-803.

Coles, D. (1956). *The law of the wake in the turbulent boundary layer*. J. Fluid Mech., 1(2), 191-226.

Coles, D. and Hirst, E. A. (1968). *Computation of turbulent boundary layers*. 1968 AFOSR-IFP Stanford Conference, Vol. II, Stanford University, CA.

Comte-Bellot, G. (1963). *Contribution a l'Etude de la Turbulence de Conduite*. PhD Thesis, University of Grenoble, France.

Craft, T. J., Launder, B. E. and Suga, K. (1993). *Extending the applicability of eddy viscosity models through the use of deformation invariants and nonlinear elements*. 5th IAHR Conf. on Refined Flow Modeling and Turbulence Measurements, Presse Ponts et Chaussées, 125-132, Paris.

Craft, T. J., Launder, B. E. and Suga, K. (1996). *Development and application of a cubic eddy-viscosity model of turbulence*. Int. Journal of Heat and Fluid Flow, 17(2), 108-115.

Daly, B. J. and Harlow, F. H. (1970). *Transport equations in turbulence*. Phys. of Fluids, 13, 2634-2649.

Davidson, L. (2010). *Fluid Mechanics, Turbulent Flow and Turbulence Modeling*. Dept. Applied Mechanics, Chalmers University of Technology, Göteborg, Sweden.

Degani, D. and Schiff, L. B. (1986). *Computation of turbulent supersonic flows around pointed bodies having crossflow separation*. Journal of Computational Physics, 66(1), 173-196.

Demuren, A. O. and Rodi, W. (1984). *Calculation of turbulence-driven secondary motion in non-circular ducts*. J. Fluid Mech., 140, 189-220.

Demuren, A. O. (1991). *Calculation of turbulence-driven secondary motion in ducts with arbitrary cross section*. AIAA Journal, 29(4), 531-537.

Donaldson, C. du P. (1972). *Construction of a Dynamic Model of the Production of Atmospheric Turbulence and the Dispersal of Atmospheric Pollutants*. ARAP Report 175, Aeronautical Research Associates of Princeton, Princeton, NJ.

Durbin, P. A. (1991). *Near wall turbulence modeling without damping function*. J. Theor. and Comput. Fluid Dynamics, 3, 1-13.

Emmons, H. W. (1954). *Shear flow turbulence*. Proc. of the 2nd U. S. Congress of Applied Mechanics, ASME.

Escudier, M. P. (1966). *The distribution of mixing-length in turbulent flows near walls*. Imp. Coll. Heat Transfer Section Rep. TWF/TN/1.

Favre, A., Kovasznay, L. S. G., Dumas, R., Gaviglio, J. and Coantic, M. (1976). *La Turbulence En Mécanique Des Fluides. Bases Théoriques Et Expérimentales*. Gauthier-Villars, Paris.

Fletcher, C. A. J. (1991). *Computational Techniques for Fluid Dynamics*. Vols. I and II, Springer-Verlag, Berlin.

Gatski, T. B. and Speziale, C. G. (1993). *On explicit algebraic stress models for complex turbulent flows*. Journal of Fluid Mechanics, 254, 47-83.

Germano, M., Piomelli, U., Moin, P. and Cabot, W. (1990). *A Dynamic subgrid-scale eddy viscosity model*. Proc. 1990 Summer Program, Center for Turbulence Research, Stanford Univ., CA.

Germano, M., Piomelli, U., Moin, P. and Cabot, W. (1991). *A Dynamic subgrid-scale eddy viscosity model*. Physics of Fluids A, 3(7), 1760-1765.

Gibson, M. M. and Launder, B. E. (1978). *Ground effects on pressure fluctuations in the atmospheric boundary layer*. J. Fluid Mechanics, 86(3), 491-511.

Girimaji, S. S. (1996). *Fully-explicit and self-consistent algebraic Reynolds stress model*. Theoretical and Computational Fluid Dynamics, 8(6), 387-402.

Glushko, G. (1965). *Turbulent boundary layer on a flat plate in an incompressible fluid*. Izvestia Akademiya Nauk SSSR, Mekh., No. 4, p. 13.

Gümüş V., Aköz, M. S. and Kırkgöz, M. S. (2013). *Experimental and numerical modeling of submerged hydraulic jump downstream of a sluice gate*. İMO Teknik Dergi, Yazı 399, 6379-6397.

Gümüş, V., Soydan, N. G., Şimşek, O., Kırkgöz, M. S. and Aköz, M. S. (2015). *Experimental and numerical modeling of flow over a spillway*. Proc. of the 2nd Int. Conf. on Advances in Civil, Structural and Construction Eng. CSCE 2015, 55-59.

Gümüş, V., Şimşek, O., Soydan, N. G., Aköz, M. S. and Kırkgöz, M. S. (2015). *Numerical modeling of submerged hydraulic jump from a sluice gate*. Journal of Irrigation and Drainage Engineering, ASCE, ISSN 0733-9437/04015037(11).

Güzel, H. (1991). *Investigation of Energy Loss in a Dam Spillway Channel*. MSc Thesis (in Turkish), Çukurova University, Adana, Turkey.

Hanjalic, K. (2005). *Will RANS survive LES? A view of perspectives*. J. Fluids Eng. 127, 831-839.

Hanjalic, K. and Launder, B. E. (1972). *A Reynolds stress model of turbulence and its application to thin shear flow*. J. Fluid Mech., 52(4), 609-625.

Hanjalic, K. and Launder, B. E. (1976). *Contribution towards a Reynolds stress closure for low Reynolds number turbulence*. J. Fluid Mech., 74, 593-610.

Hanjalic, K., Popovac, M. and Hadziabdic, M. (2004). *A robust near-wall elliptic relaxation eddy-viscosity turbulence model for* CFD. Int. J. Heat and Fluid Flow, 25, 1047-1051.

Hinze, J. O. (1975). *Turbulence*. 2nd ed., McGraw-Hill, New York, NY.

Hirt, C. V. and Nichols, B. D. (1981). *Volume of fluid method for the dynamics of free boundaries*. Journal of Computational Physics, 39(1), 201-225.

Horiuti, K. (1990). *Higher-order terms in the anisotropic representation of Reynolds stresses*. Physics of Fluids A: Fluid Dynamics, 2, 1708.

Issa, R. I. (1986). *Solution of the implicitly discretized fluid flow equations by operator-splitting*. J. Comput. Phys., 62, 40-65.

Jones, W. P. and Launder, B. E. (1972). *The prediction of laminarization with a two-equation model of turbulence*. Int. Journal of Heat and Mass Transfer, 15, 301-314.

Jongen, T. (1992). *Simulation and Modeling of Turbulent Incompressible Flows*. PhD thesis, EPF Lausanne, Lausanne, Switzerland.

Kato, M. and Launder, B. E. (1993). *The modeling of turbulent flow around a stationary and vibrating square cylinder*. Proc. 9th Symp. on Turbulent Shear Flows, Kyoto, Japan, Vol. 9, pp. 10.4/1-10.4/6.

Kırkgöz, M. S. (1989). *Turbulent velocity profiles for smooth and rough open channel flow*. J. Hyd. Eng., ASCE, 115(11), 1543-1561.

Kırkgöz, M. S. and Ardıçlıoğlu, M. (1997). *Velocity profiles of developing and developed open channel flow*. J. Hyd. Eng., ASCE, 123(12), 1099-1105.

Kırkgöz, M. S., Aköz, M. S. and Öner, A. A. (2009). *Numerical modeling of flow over a chute spillway*. Journal of Hydraulic Research, 47(6), 790-797.

Kırkgöz, M. S., Aköz, M. S., Gümüş, V. and Öner, A. A. (2011). *Experimental and numerical modeling of flow over a gate-controlled broad-crested weir*. Proc. 5th National Water Engineering Symposium, 12-16 Sept., İstanbul, 611-624.

Kırkgöz, M. S., Gümüş, V., Soydan, N. G., Şimşek, O. and Aköz, M. S. (2012). *Experimental and numerical modeling of flow over a gate-controlled semicylinder weir*. 10th Int. Congress on Advances in Civil Engineering, 17-19 October, METU, Ankara, 0184, 1-10.

Kim, J., Moin, P. and Moser, R. (1987). *Turbulence statistics in fully developed channel flow at low Reynolds number*. J. Fluid Mech., 177, 133-166.

Kim, W. W. and Menon, S. (1994). *A new dynamic one equation subgrid scale model for large eddy simulations*. AIAA Paper No. 95-0356.

Klebanoff, P. S. (1954). *Characteristics of turbulence in a boundary layer with zero pressure gradient*. NACA Report 3178, Washington, DC.

Klebanoff, P. S. (1955). *Characteristics of turbulence in a boundary layer with zero pressure gradient*. NACA Report 1247, Washington, DC.

Kline, S. J., Morkovin, M. V., Sovran, G. and Cockrell, D. J. (1969). *Computation of turbulent boundary layers*. 1968 AFOSR-IFP Stanford Conference, Vol. I, Stanford University, CA.

Kok, J. C., Dol, H. S., Oskam, B. and van der Ven, H. (2004). *Extra-large eddy simulation of massively separated flows*. AIAA paper 2004-264, Reno, NV.

Kolmogorov, A. N. (1941). *Local structure of turbulence in incompressible viscous fluid for very large Reynolds number*. Doklady Akademiya Nauk SSSR, 30, 299-303.

Kolmogorov, A. N. (1942). *Equations of turbulent motion of an incompressible fluid*. Izvestia Academy of Sciences, USSR; Physics, 6 (1 and 2), 56-58.

Kraichnan, R. H. (1991). *Turbulent cascade and intermittency growth*. Proc. Royal Society of London, Mathematical and Physical Sciences, A434 (1890), 65-78.

Lam, C. K. G. and Bremhorst, K. (1981). *A modified form of the* k-ε *model for predicting wall turbulence*. J. of Fluid Engng., 103(3), 456-460.

Landahl, M. T. and Mollo-Christensen, E. (1992). *Turbulence and Random Processes in Fluid Mechanics*. Second Ed., Cambridge University Press, New York, NY.

Langtry, R. B. and Menter, F. R. (2009). *Correlation-based transition modeling for unstructured parallelized computational fluid dynamics codes*. AIAA Journal, 47(12), 2894-2906.

Laufer, J. (1951). *Investigation of Turbulent Flow in a Two-Dimensional Channel*. NACA Report 1053.

Launder, B. E. and Jones, W. P. (1969). *A note on Bradshaw's hypothesis for laminarization*. ASME Paper 69-HT-12.

Launder, B. E. and Spalding, D. B. (1972). *Mathematical Models of Turbulence*. Academic Press, London.

Launder, B. E. and Sharma, B. I. (1974). *Application of the energy dissipation model of turbulence to the calculation of flow near a spinning disc*. Letters in Heat and Mass Transfer, 1(2), 131-138.

Launder, B. E. and Spalding, D. B. (1974). *The numerical computation of turbulent flows*. Computer Methods in Applied Mechanics and Energy, 3, 269-289.

Launder, B. E., Reece, G. J. and Rodi, W. (1975). *Progress in the development of a Reynolds stress turbulence closure*. J. Fluid Mech., 68(3), 537-566.

Launder, B. E. (1989). *Second moment closure: present and future*. Int. J. Heat and Fluid Flow, 10(4), 282-300.

Launder, B. E. and Shima, N. (1989). *Second moment closure for the near wall sublayer: development and application*. AIAA Journal, 27(10), 1319-1325.

Launder, B. E. (1993). *Modelling convective heat transfer in complex turbulent flows*. Proc. of the 2nd Int. Symp. on Engineering Turbulence Modelling and Measurements, Florence, Italy, 3-22.

Leonard, A. (1974). *Energy cascade in large-eddy simulations of turbulent fluid flows*. Advances in Geophysics, 18A, 237-248.

Libby, P. A. (1996). *Introduction to Turbulence*. Taylor and Francis, Bristol, PA.

Lien, F. S. and Leschziner, M. A. (1994). *Assessment of turbulence-transport models including non-Linear* RNG *eddy-viscosity formulation and second-moment closure for flow over a backward-facing step*. Computers and Fluids, 23, 983-1004.

Lien, F. S., Chen, W. L. and Leschziner, M. A. (1996). *Low-Reynolds number eddy-viscosity modelling based on non-linear stress-strain/vorticity relations*. Proc. 3rd Symp. on Engineering Turbulence Modelling and Measurements, Crete, Greece.

Lilly, D. K. (1966). *On the Application of the Eddy Viscosity Concept in the Inertial Subrange of Turbulence*. NCAR Manuscript 123.

Lilly, D. K. (1992). *A proposed modification of the Germano subgrid scale closure model*. Phys. of Fluids A, 4(3), 633-635.

Lindgren, E. R. (1965). *Experimental study on turbulent pipe flows of distilled water*. Civil Eng. Dept., Oklahoma State University, Rept. IAD621071.

Lumley, J. L. (1970). *Toward a turbulent constitutive equation*. Journal of Fluid Mechanics, 41, 413-434.

Lumley, J. L. and Newman, G. R. (1977). *The return to isotropy of homogeneous turbulence*. J. Fluid Mech., 82(1), 161-178.

Lumley, J. L. (1978). *Computational modeling of turbulent flows*. Adv. Appl. Mech., 18, 123-176.

Mellor, G. L. and Gibson, D. M. (1966). *Equilibrium turbulent boundary layers*. J. Fluid Mechanics, 24, Part 2, 225-253.

Mellor, G. L. and Herring, H. J. (1973). *A survey of mean turbulent field closure models*. AIAA Journal, 11(5), 590-599.

Menter, F. R. (1994). *Two-equation eddy-viscosity turbulence models for engineering applications*. AIAA Journal, 32(8), 1598-1605.

Menter, F. R., Kuntz, M. and Langtry, R. (2003). *Ten years of industrial experience of the* SST *turbulence model*. In: Hanjalic, K., Nagano, Y. and Tummers, M. J. (Eds.), Turbulence, Heat and Mass Transfer 4, 624-632, New York, Wallingford (UK).

Menter, F. R., Langtry, R. B., Likki, S. R., Suzen, Y. B., Huang, P. G. and Volker, S. (2004). *A Correlation Based Transition Model Using Local Variables, Part 1 - Model Formulation*. ASME-GT2004-53452.

Moser, R. D. and Moin, P. (1984). *Direct Numerical Simulation of Curved Turbulent Channel Flow*. NASA TM-85974.

Naot, D. and Rodi, W. (1982). *Numerical simulations of secondary currents in channel flow*. J. Hyd. Div., ASCE, 108(HY8), 948-968.

Nezu, I. and Nakagawa, H. (1993). *Turbulence in Open-Channel Flows*. A. A. Balkema Publishers, Rotterdam, Netherlands.

Nicoud, F. and Ducros, F. (1999). *Subgrid-scale stress modelling based on the square of the velocity gradient tensor*. Flow, Turbulence and Combustion, 62(3), 183-200.

Nikuradse, J. (1933). *Stromungsgesetze in rauhen Rohren*. VDI-Forschungsheft, 361.

Oberkampf, W. L. and Trucano, T. G. (2002). *Verification and validation in computational fluid dynamics*. Prog. Aerosp. Sci., 38, 209-272.

Oner, A. A., Aköz, M. S., Kırkgöz, M. S. and Gümüş, V. (2012). *Experimental validation of volume of fluid method for a sluice gate flow*. Advances in Mechanical Engineering, Article ID 461708.

Park, T. S. and Sung, H. J. (1997). *A new low-Reynolds number* k-ε-f_μ *model for predictions involving multiple surfaces*. Fluid Dynamics Research, 20, 97-113.

Patankar, S. V. (1980). *Numerical Heat Transfer and Fluid Flow*. Hemisphere Publishing Corp., Taylor & Francis Group, New York.

Patankar, S. V. and Spalding, D. B. (1970). *Heat and Mass Transfer in Boundary Layers, A General Calculation Procedure*. Intertext Books, London.

Patankar, S. V. and Spalding, D. B. (1972). *A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows*. Int. J. Heat Mass Transfer, 15(10), 1787-1806.

Patel, V. C., Rodi, W. and Scheuerer, G. (1985). *Turbulence models for near-wall and low Reynolds number flows: A review*. AIAA Journal, 23(9), 1308-1319.

Patel, V. C. (1988). *Perspective: flow at high Reynolds number and over rough surfaces-achilles heel of CFD*. J. Fluids Engng., 120(3), 434-444.

Prandtl, L. (1904). *Über Flüssigkeitsbewegung bei sehr Kleiner Reibung*. Proc. Third Internat. Math. Cong. Heidelberg.

Prandtl, L. (1925). *Über die ausgebildete Turbulenz.* ZAMM 5, 136-139.

Prandtl, L. (1933). *Neuere Ergebnisse der Turbulenzforschung*. Z. VDI 77, 105-114.

Prandtl, L. (1945). *Über ein neues Formelsystem für die ausgebildete Turbulenz*. Nachr. Akad. Wiss. Göttingen, Math-Phys. Kl. II., 6-19.

Raudkivi, A. J. and Callander, R. A. (1975). *Advanced Fluid Mechanics*. Edward Arnold Publishers Ltd., London.

Reynolds, O. (1883). *An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the laws of resistance in parallel channels*. Phil. Trans. Royal Society of London, A174, 935-982.

Reynolds, O. (1895). *On the dynamical theory of incompressible viscous fluids and the determination of the criterion*. Philosophical Transactions of the Royal Society of London, Series A, 186, 123-164.

Reynolds, W. C. (1987). *Fundamentals of Turbulence for Turbulence Modeling and Simulation*. In: Lecture Notes for von Karman Institute, AGARD Lecture Series No. 86, 1-66, New York: NATO.

Richardson, L. F. (1910). *The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam*. Phil. Trans. Roy. Soc. London, Ser. A, 210, 307-357.

Richardson, L. F. and Gaunt, J. A. (1927). *The deferred approach to the limit*. Phil. Trans. Roy. Soc. London, Ser. A, 226, 299-361.

Rodi, W. (1976). *A new algebraic relation for calculating the Reynolds stresses*. ZAMM, 56, 219-221.

Rodi, W. (1980). *Turbulence models and their application in hydraulics*. Conf. IARH-Section on Fundamentals of Division II. Exp. and Math. Fluid Dynamics, Delft.

Roache, P. J. (1997). *Quantification of uncertainty in computational fluid dynamics*. Annual Review of Fluid Mechanics, 29, 123-160.

Roache, P. J. (1998). *Verification of codes and calculations*. AIAA Journal, 36(5), 696-702.

Rotta, J. C. (1951). *Statistische Theorie nichthomogener Turbulenz*. Zeitschrift für Physik. 129, 547-572.

Rotta, J. C. (1956). *Experimenteller Beitrag zur Entstehung turbulenter Stromungen im Rohr*. Ing.-Arch., 24, 258-281.

Rotta, J. C. (1968). *Über eine Methode zur Berechnung turbulenter Scherströmungen*. Aerodynamische Versuchanstalt Göttingen, Rep. 69 A 14.

Rubinstein, R. and Barton, J. M. (1990). *Nonlinear Reynolds stress models and renormalization group*. Physics of Fluids, A2(8), 1472-1476.

Saffman, P. G. (1976). *Development of a complete model for the calculation of turbulent shear flows*. Symposium on Turbulence and Dynamical Systems, Duke Univ., Durham, NC.

Schiestel, R. (2008). *Modeling and Simulation of Turbulent Flows*. ISTE Ltd. and John Wiley & Sons, Inc., London.

Schlichting, H. (1933). *Laminare Strahlenausbreitung*. Z. Angew Math. Mech., 13, 260-263.

Schumann, U. (1977). *Realizability of Reynolds-stress turbulence models*. The Physics of Fluids, 20(5), 721-725.

Shaanan, S., Ferziger, J. H. and Reynolds, W. C. (1975). *Numerical Simulation of Turbulence in the Presence of Shear*. Report No. TF-6, Dept. of Mech. Eng., Stanford University, CA.

Shih, T. H. and Lumley, J. L. (1993). *Critical comparison of second order closures with direct numerical simulations of homogeneous turbulence*. AIAA Journal, 31(4), 663-670.

Shih, T. H., Zhu, J. and Lumley, J. L. (1994). *A New Reynolds Stress Algebraic Equation Model*. NASA Technical Memorandum 106644.

Shih, T. H., Zhu, J. and Lumley, J. L. (1995). *A new Reynolds stress algebraic equation model*. Computer Methods in Applied Mechanics and Engng., 125(1-4), 287-302.

Shih, T. H., Liou, W. W., Shabbir, A., Yang, Z. and Zhu, J. (1995). *A new* k-ε *eddy viscosity model for high Reynolds number turbulent flows*. Computers and Fluids, 24(3), 227-238.

Smagorinsky, J. (1963). *General circulation experiments with the primitive equations. I.The basic experiment*. Mon. Weather Rev., 91, 99-164.

Smith, A. M. O. and Cebeci, T. (1967). *Numerical Solution of the Turbulent Boundary-Layer Equations*. Douglas Aircraft Division Report DAC 33735.

So, R. M. C., Lai, Y. G., Zhang, H. S. and Hwang, B. C. (1991). *Second-order near-wall turbulence closures: a review*. AIAA Journal, 29(11), 1819-1835.

Soydan-Oksal, N. G., Aköz, M. S. and Şimşek, O. (2020). *Numerical modeling of trapezoidal weir flow with* RANS*,* LES *and* DES *models*. Sadhana, 45:91, Indian Academy of Sciences.

Spalart, P. R. and Allmaras, S. R. (1992). *A one-equation turbulence model for aerodynamic flows*. AIAA Paper 92-439, Reno, NV.

Spalart, P. R., Jou, W. H., Strelets, M. and Allmaras, S. R. (1997). *Comments on the feasibility of* LES *for wings, and on a hybrid* RANS/LES *approach*. Proc. 1st AFOSR Int. Conf. on DNS and LES, Louisiana Tech., Greyden Press, Columbus, OH.

Spalding, D. B. (1961). *A single formula for the law of the wall*. J. Appl. Mech., E28, 455-458.

Spalding, D. B. (1969). *The calculation of the Length Scale of Turbulence in Some Shear Flows Remote from Walls*. Progress in Heat and Mass Transfer, Vol. 2, Pergamon Press London.

Speziale, C. G. (1987). *On non-linear* k-l *and* k-ε *models of turbulence*. J. Fluid Mech., 178, 459-475.

Speziale, C. G., Abid, R. and Anderson, E. C. (1990). *A critical evaluation of two-equation models for near wall turbulence*. AIAA Paper 90-1481, Seattle, WA.

Speziale, C. G. (1991). *Analytical methods for the development of Reynolds stress closures in turbulence*. Annual Review of Fluid Mechanics, 23, 107-157.

Speziale, C. G., Sarkar, S. and Gatski, T. B. (1991). *Modeling the pressure-strain correlation of turbulence*: *An invariant dynamic system approach*. J. Fluid Mech., 227, 245-272.

Speziale, C. G., Gatski, T. B. and Sarkar, S. (1992). *On testing models for the pressure-strain correlation of turbulence using direct simulations*. Phys. of Fluids A, 4(12), 2887-2899.

Sümer, B. M. (2007). *Lecture Notes on Turbulence*. Technical University of Denmark, MEK, Coastal, Maritime & Structural Engineering Section, Lyngby, Denmark.

Şimşek, O., Soydan, N. G., Gümüş, V., Aköz, M. S. and Kırkgöz, M. S. (2015). *Numerical modeling of B-type hydraulic jump at an abrupt drop*. İMO Teknik Dergi, Yazı 441, 7215-7240.

Şimşek, O., Aköz, M. S. and Soydan, N. G. (2016). *Numerical validation of open channel flow over a curvilinear broad-crested weir*. Progress in Computational Fluid Dynamics, 16(6), 364-378.

Taylor, G. I. (1935). *Statistical theory of turbulence*. Proc. Royal Society of London, A151, 421-435.

Taylor, G. I. (1938). *The spectrum of turbulence*. Proc. Royal Society of London, Mathematical and Physical Sciences, A164, 476-490.

Tennekes, H. and Lumley, J. L. (1972). *A First Course in Turbulence*. MIT Press, Cambridge, Massachusetts.

Tessicini, F., Li, N. and Leschziner, M. A. (2005). *Zonal* LES/RAN *modelling of separated flow around a three-dimensional hill*. ERCOFTAC Workshop Direct and Large-Eddy Simulation 6, Poitiers.

Thompson, B. and Whitelaw, J. (1985). *Characteristics of a trailing-edge flow with turbulent boundary-layer separation*. J. Fluid Mech., 157, 305-326.

Tollmien, W. (1931). *Grenzschichttheorie*. In: *Handbuch der Experimentalphysik* IV,1, Leipzig, 239–287.

Travin, A. K., Shur, M., Strelets, M. and Spalart P. R. (2002). *Physical and numerical upgrades in the detached-eddy simulations of complex turbulent flows*. In: Friederich, R. and Rodi, W. (Eds.), Fluid Mechanics and its Applications, Advances in LES of Complex Flows, 239-254, Academic Publishers, Dordrecht.

van Doormal, J. P. and Raithby, G. D. (1984). *Enhancements of the* SIMPLE *method for predicting incompressible fluid flows*. Numerical Heat Transfer, 7, 147-163.

van Driest, E. R. (1956). *On turbulent flow near a wall*. J. Aero. Sci., 23, 1007-1011.

Versteeg, H. K. and Malalasekera, W. (2007). *An Introduction to Computational Fluid Dynamics*. Pearson Education Ltd., London.

von Karman, T. (1930). *Mechanische Ahnlichkeit und Turbulenz*. Nah. Ges. Wiss. Göttingen, Math. Phys. Klasse, 58.

von Karman, T. (1934). *Turbulence and skin friction*. J. Aeronautical Sciences, 1(1), 1-20.

von Karman, T. and Howarth, L. (1938). *On the statistical theory of isotropic turbulence*. Proc. Royal Society of London, Mathematical and Physical Sciences, A164 (917), 192-215.

Walters, D. K. and Cokljat, D. (2008). *A three-equation eddy-viscosity model for Reynolds-averaged Navier-Stokes simulations of transitional flows*. Journal of Fluids Engineering, 130.

White, F. M. (1991). *Viscous Fluid Flow*. McGraw-Hill, Inc., New York.

Wilcox, D. C. (1988a). *Reassessment of the scale-determining equation for advanced turbulence models*. AIAA Journal, 26(11), 1299-1311.

Wilcox, D. C. (1988b). *Multiscale model for turbulent flows*. AIAA Journal, 26(11), 1311-1320.

Wilcox, D. C. (1994). *Simulation of transition with a two-equation turbulence model*. AIAA Journal, 32(2), 247-255.

Wilcox, D. C. (1998). *Turbulence Modeling for* CFD. DCW Industries, Inc., La Canada, California.

Wilcox, D. C. (2000). *Turbulence Modeling for* CFD. KNI, Inc., Anaheim, California.

Wolfshtein, M. (1967). *Convection Processes in Turbulent Impinging Jets*. Imperial College, Heat Transfer Section Report SF/R/2, London.

Wolfshtein, M. (1969). *The velocity and temperature distribution in one-dimensional flow with turbulence augmentation and pressure gradient*. Int. J. Heat and Mass Transfer, 12, 301-318.

Yakhot, V. and Orszag, S. A. (1986). *Renormalization group analysis of turbulence: 1. Basic theory*. Journal of Scientific Computing, 1, 3-51.

Yakhot, V., Orszag, S. A. and Panda, R. (1989). *Computational test of renormalization group theory of turbulence*. C. C. Chau et al. (eds.), Recent Advances in Computational Fluid Dynamics, Springer-Verlag Berlin, Heidelberg.

Yakhot, V., Orszag, S. A., Thangam, S., Gatski, T. B. and Speziale, C. G. (1992). *Development of turbulence models for shear flows by a double expansion technique*. Physics of Fluids A, 4, 1510-1520.

Yan, J., Mocket, C. and Thiele, F. (2005). *Investigation of alternative length scale substitutions in detached-eddy simulation*. Flow, Turbulence and Combustion, 74, 85-102.

Yap, C. (1987). *Turbulent Heat and Momentum Transfer in Recirculating and Impinging Flows*. PhD Thesis, University of Manchester and UMIST, Dept. of Mech. Eng. Report TFD/87/1.

Zeierman, S. and Wolfshtein, M. (1986). *Turbulent time scale for turbulent flow calculations*. AIAA Journal, 24(10), 1606-1610.

INDEX

Acceleration and deceleration effect on RSM, 308
Adapted grid, 358
Algebraic stress models, 273
Aspect ratio of cells, 359
Autocorrelation, 80

BALDWIN-BARTH model, 199
BALDWIN-LOMAX model, 189
Baseline k-ω model, 251
Basic principles of fluid flow, 362
BSL model equations, 253
Blending function, 251
Boundary conditions, 341
Boussinesq hypothesis, 135
B-type hydraulic jump at an abrupt drop, 376
Buoyancy-driven turbulent flows, 114
Buoyant production, 293

CEBECI-SMITH model, 187
Cell types, 351
CFD code structure, 2
Characteristic features of turbulence, 50
Classes of turbulence models, 183
Classification of fluid flow equations, 46
CLEAR-VOF algorithm, 365
Closure problem, 181
Compressed mesh, 358
Conservation laws, 21
Conservation of energy, 36
Conservation of mass, 22, 119
Conservation of momentum, 25, 120
Constitutive equations, 14
Convective motion of fluid, 6
Correlation coefficient, 64
Correlations of turbulence, 72
Courant number, 311, 366
CRAFT-LAUNDER-SUGA model, 272
Cubic stress-strain relation models, 272
Curvature effects on RSM, 306

Damping effect, 282
Damping function, 258
Damping function approach, 328
Description of flow field, 5
Detached eddy simulation, 329
Diffusivity of turbulence, 54
Direct numerical simulation, 309
Discretization errors, 340
Dissipative range, 86
Dissipation term, 299
Domain geometry, 341
DURBIN model, 281
Durbin's realizable constraint, 208
Dynamic kinetic energy SGS model, 327
Dynamic SMAGORINSKY-LILLY model, 324

Effective turbulent viscosity, 101
Elliptic equation for relaxation function, 285
Energy dissipation rate, 94
Energy production range, 93
Enhanced near-wall treatment, 230
Equilibrium problems, 43
Equilibrium turbulent flows, 111
Error band, 348
Error estimates, 347
Errors and uncertainties, 335
Eulerian autocorrelation, 81
Eulerian time scales, 81
Exact K-transport equation, 170
Exact k-transport equation, 159
Exact RSTEs, 175, 176
Exact ε-transport equation, 169
Explicit algebraic stress models, 276

Favre averaging, 117
Filter function, 316
Flow over a broad-crested weir, 367
Flow over a curvilinear broad-crested weir, 408
Flow over a gate-controlled broad-crested weir, 372
Flow over a gate-controlled semicylinder weir, 403

Flow over a semicylinder weir, 393
Flow over a spillway, 422
Flow work, 40
Fluid properties, 342
Fourier series, 89

GATSKI-SPEZIALE model, 277
Gaussian distribution, 62
Gaussian filter, 317
General form of z-equation, 203
General transport equations, 29
GEO-RECONSTRUCT scheme, 365
GIRIMAJI model, 278
Grid convergence index, 345
Grid density, 357
Grid design, 351
Grid types, 352

HANJALIC ζ-f model, 284
Heat transfer, 15, 18
Heat transport, 38
History of turbulence modeling, 181
Homogeneous turbulence, 71

Implicit algebraic stress models, 274
Inertial range, 86, 93
Initial and boundary conditions, 47
Injective energy containing range, 86
Inner region, 108
Input documentation, 360
Input uncertainties, 335, 341
Instantaneous kinetic energy, 119
Instantaneous strain-rate tensor, 118
Instantaneous stress tensor, 118
Intermittency factor, 68
Interpretation of computational results, 361
Isotropic turbulence, 71, 73, 76
Iterative convergence errors, 336

JOHNSON-KING model, 192

Karman-Howarth equation, 80
Kato-Launder modification, 206
Kinetic energy of mean flow, 147, 151
Kolmogorov scales, 87, 100
Kolmorov spectrum law, 95
Kurtosis of pdf, 66
k-ε models, 209
k-ω models, 243

Lagrangian autocorrelation, 82
Lagrangian time scales, 83
Laminar boundary layer, 56
Laminar flow, 21
Large eddy Reynolds number, 86
Large eddy simulation, 313
Large-scale turbulent viscosity, 258
Large turbulent eddies, 85
Largest permanent eddies, 91
Launder-Sharma model, 226
Lam-Bremhorst model, 227
Law of the wake, 110
Law of the wall, 108
Length scales in turbulent flows, 56, 75
Lewis number, 17
LIEN-CHEN-LESCHZINER model, 271
Linear constitutive relation, 133
Linear eddy viscosity models, 186
Low Reynolds number models, 222
LRR linear pressure-strain model, 295
LRR wall-reflection term, 298
LUMLEY model, 265

Macro time scale, 81
Marching problems, 43
Mass transfer, 15, 16
Mean product of velocity fluctuations, 67
Mean velocity, 64
Menter k-ω models, 250
Mesh naming, 352
Mesh quality, 356
Microscales, 77
Micro time scale, 82
Mixing-length models, 186
Modeled k-transport equation, 166, 241
Modeled Reynolds equation, 301
Modeled ε-transport equation, 169, 241
Modeling the scalar dissipation rate, 302
Modeling the effective viscosity, 234
Modeling turbulence kinetic energy, 301
Modeling the turbulent viscosity, 240, 302
Modified k-ω model, 246
Modified turbulent viscosity, 215
Molecular diffusion, 292
Momentum transfer, 15, 17
Momentum transport equations, 28, 127
Multi-block structured grid, 355

Navier-Stokes equations, 27
Near-wall dissipation, 259

Near-wall treatment, 218, 245
Near-wall treatment for k-ω models, 250
Near-wall treatment for LES models, 328
Nonlinear constitutive relation models, 263
Nonlinear eddy viscosity models, 263
Nonlinear k-ε models, 264
Numerical errors, 336

One-equation models, 195
Outer region, 109

PARK-SUNG model, 283
Parseval's formula, 89
Physical model uncertainties, 335, 343
Postprocessor, 4
Prandtl's mixing-length hypothesis, 135
PRANDTL model, 196
Prandtl number, 19, 132
Prandtl-Schmidt number, 137
Preliminary concepts, 1
Preprocessor, 2
Pressure fluctuations, 293
Pressure-strain correlation, 293, 305
PRESTO algorithm, 366
Probability density function, 60, 61
Production term, 292

Quadratic stress-strain relation models, 265
Quality checking of computed results, 361

Random nature of turbulence, 53
RANS-based turbulence models, 185
RANS equations, 122
Rapid distortion constraint, 269
Rapid pressure-strain term, 297
Rayleigh number, 116
Realizability, 208, 240
Realizability constraint, 269
Realizable k-ε based DES model, 333
Realizable k-ε model, 236
Refined structured grid, 353
Relation between viscosity and vorticity, 31
Renormalized group k-ε model, 233
Reynolds averaging, 118
Reynolds stresses, 124, 134
Reynolds stress models, 285
Reynolds stress tensor, 275, 277, 304
Reynolds transport theorem, 21
Richardson's energy cascade, 85
Richardson number, 115
RNG-LES model, 326
RNG swirl modification, 234
RODI model, 274
Rotation rate tensor, 7
Roundoff errors, 336
RSM-ε, 287
RSM-ω, 304
RSTEs, 288
RUBINSTEIN-BARTON model, 268

SAFFMAN model, 266
Scale relations of turbulent eddies, 100
Schmidt number, 17, 132
Schwarz inequality, 208, 240, 269
Semiempirical modeling, 59
Separation-induced transition, 261
SGS stresses, 320
SGS stress term, 321
Shear driven turbulent flows, 113
Shear stress transport k-ω model, 254
SHIH-ZHU-LUMLEY model, 268
SIMPLE algorithm, 366
Single-point correlation, 72
Skewness of cells, 359
Skewness of pdf, 65
SMAGORINSKY-LILLY model, 322
Small scales in turbulence, 58
Smoothness of cells, 359
Solver, 3
Space correlation, 73
SPALART-ALLMARAS based DES model, 331
SPALART-ALLMARAS model, 200
Spalding's law of the wall, 109
Specific dissipation rate ω, 216
Specific dissipation rate equation, 244, 247, 304
Spectral analysis of turbulence energy, 90
SPEZIALE k-τ model, 242
SPEZIALE model, 266
SSG quadratic pressure-strain model, 298
SST k-ω based DES model, 333
Standard k-ε model, 210
Standard k-ε based DES model, 332
Standard k-ω model
State of stress, 5
Strain rate Tensor, 6
Stretched mesh, 357

Structured curvilinear grids, 354
Structured grid, 353
Subgrid-scale models, 321
Submerged hydraulic jump downstream of a drum gate, 417
Submerged hydraulic jump from a Sluice gate, 385
System rotation term, 300

Taylor microscale, 83, 96
The v'^2-f models, 281
Time-averaged continuity equation, 120
Time-averaged total kinetic energy, 149
Time averaging of a variable, 117
Time correlation, 80
Tollmien-Schlichting waves, 70
Total stress in turbulent flow, 124
Transition k-k_l-ω model, 257
Transition k-ω models, 257
Transition SST model, 260
Transport equation for intermittency, 260
Transport equation for Re-stresses, 274
Transport equations for k, 212, 275
Transport equations for ε, 212
Transport properties of a fluid, 15
Trapezoidal weir flow with RANS based, LES and DES models, 427
Turbulence energy cascade, 93
Turbulence energy spectrum, 88, 91
Turbulence in fluid flow, 49
Turbulence intensity, 216
Turbulence kinetic energy, 87, 88, 93, 149, 154, 244, 247
Turbulence length scale, 217, 285
Turbulence modeling, 181
Turbulence time scale, 285
Turbulent boundary layer, 57
Turbulent cascade, 85, 95
Turbulent diffusion, 85, 290
Turbulent equilibrium, 111
Turbulent heat transfer, 130
Turbulent mass transfer, 129
Turbulent momentum transfer, 130
Turbulent Reynolds number, 101
Turbulent Reynolds number equation, 199
Turbulent viscosity, 102, 284
Two-equation models, 201
Two-layer model, 228
Two-point correlation, 71

Universal equilibrium theory, 88, 94
Unstructured grid, 355

Validation, 344
Validation process of CFD modeling, 349
van Driest mixing-length hypothesis, 139
Variance, 65
Velocity correlation, 132
Velocity-defect law, 109
Velocity profiles, 106
Verification, 344
Viscous wall law, 108
Volume-average box filter, 317
Volume of fluid method, 363
von Karman's similarity hypothesis, 138
Vortex stretching, 34, 104, 105
Vortex tilting, 35, 104, 105
Vorticity transport equation, 32, 104, 142
Vorticity vector, 8

Wall-adapting local eddy-viscosity model, 328
Wall boundary condition for ε, 227
Wall functions, 218, 245
Wall reflection terms, 296, 297
Wave number, 90

Yap Correction, 206

Zonal two-layer approach, 329
Zero-equation models, 186