CHAPTER 1

Mathematical Concept Development and Number Sense

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INTRODUCTION

Jean Piaget's theory is that children construct knowledge by interacting with their environment and trying to understand everything they encounter in their lives without waiting for anyone to teach them. According to the results of scientific research based on Piaget's theory of cognitive development, information about how and when mathematical concept development occurs has been obtained. Children construct knowledge through interactions with their physical environment (Cited by Charlesworth & Lind, 2010). When scrutinizing our environment, we observe that the world consists of physical order and elements. In this order, children continue their conceptual development by interacting with and exploring structures, but some mathematical concepts cannot be learned by discovering these existing elements through direct observation (Wadsworth, 2004). For example, reversibility-related items and situations are few in our environment; therefore, reversibility is a feature that the child himself should reveal.

Concepts such as having an egocentric point of view, focusing, reversing, and limited reasoning that Piaget put forward are interrelated. The development or immaturity of these concepts explains mental development in the early years of the preoperational

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period. As cognitive development continues, all these features gradually decrease. For example, the decrease in the egocentric point of view enables the child to focus more and observe the changes (As cited in Wadsworth, 2004). These features are also natural developmental processes necessary for developing logical reasoning. We can see these features most clearly in the solutions of conservation problems (Berk, 1999). In later years, reversibility also appears before children with algebraic expressions.

Piaget states that in the one-to-one matching stage, which is the first stage in the development of the number concept, equivalent sets are formed by the children. At this stage, each object is paired with another object by the children. Some of the children perform this operation using only numbers. In daily life, one-to-one matching is seen in the questions asked by children or in the activities performed. Achieving number conservation constitutes the second step. In the case of the development of number conservation, the child is aware that the two numbers do not change, even if the way of grouping changes. However, it can be said that number conservation develops if the child expresses that the two groups are the same in case the location or frequency of the objects changes (Altun, 2011). In other words, the idea that the result does not change even if the location of the objects is changed leads us to the principle of conservation. The situation where the amount of the substance does not change even when a change is made in the size of the substance is called conservation. Conservation ability reveals the logical-mathematical structure developed by the child. It is stated that this ability usually begins to develop towards the end of the preoperational period (Wadsworth, 2004). Children reach different conservations in the concrete operational stage (quantities, length, area, volume, etc.) at different ages.

The cognitive development theory put forward by Piaget focuses on the length of the queue, but it is stated that the numerical characteristics of the objects are ignored in this theory (Smith, 2009). It is stated that the child does not focus on change during the observation of events by the child. The child in this situation does not consider the difference between the previous and subsequent situations and focuses only on the changed situation. It can also be understood from the reasoning in the answer given by the child. The child in this situation mostly gives a perceptual response (Charlesworth & Lind, 2010). If the size or order of objects changes, children perceive that their number also changes. They do not perceive that the number of objects does not change when their location changes. However, even if children accept that the number of objects does not change, they do not believe that the amount has not changed. Piaget states that the lack of this skill stems from conservation (Cited by Nelson, 2007). In conservation of quantities studies, if a 4- or 5-year-old child is asked to make another row in the same way by showing objects arranged at equal intervals, the child can form a row with the same length, but when the child creates this row, only the length of the row is taken into account, not the number of objects. The child aged 6 or 7 has the mental capacity necessary for number conservation. Simultaneously, around this age, children begin to notice changes and develop the ability to reverse processes. In other words, in this period, the child becomes aware that the number of objects will not change due to the change in the length of the row (Smith, 2009). This situation, which is observed more in the preoperational period, is replaced by reduced errors in the conservation of objects at the beginning of the concrete operational period.

For children to gain number conservation skills, they must face situations where they can compare objects regardless of their numbers. In this context, children may be requested to bring the same number of objects. In this way, children can acquire a number of conservation skills (Taşkın, 2021). For example, children were given five cars and asked, “How many different ways can you park the cars?” With a study like this, children will park cars in different ways and realize that the number of cars does not change.

DEVELOPMENT OF MATHEMATICAL CONCEPTS

When numbers are written on paper or displayed on a calculator screen, they are represented by a series of abstract symbols, and for many children, they are just wavy lines. Children go through a long and challenging learning process to associate which wave with which sound. “5” to five, “8” to eight, etc. They read the sounds in order as they learn to count a group of objects. For example, five first becomes meaningful as the number after four. In the beginning, they have no accurate understanding that five represents five multiplicities. This situation is more commonly seen as a result of an add-on to an add-on. Children encounter numbers with different meanings. For example, like four pens, row four, or going to page 4 in a book. Children play on the number lane on the playground and may find themselves standing at number 4. They learn that they are 4 years old and their next birthday will be 5. 4 must be parked in area 4, between 3 and 5. There are also places where numbers are used; sometimes, the number is used as a label or a name. For example, A bus with the number 36 means a number 36 you can get on, and it is not much different from the Çankaya bus. It simply identifies the bus and distinguishes it from buses on other routes. When we see a bus number 36, we do not expect it to be followed by bus number 37 and then number 38. Using numbers with different meanings in this way expands children’s vocabulary (Haylock & Manning, 2019; Smith, 2009; Tandi Clausen-Mayn (2005). As children learn different meanings, they also feel that numbers are used everywhere in our daily lives. At the same time, they are independent of concrete objects, in their minds. A rich abstract structure is formed regarding the concept of number.

Learning the concept of numbers means understanding what numbers mean, knowing how and where to use them, being able to count correctly and regularly, and relating numbers (Baykul, 2006). Acquiring basic number concepts is very important for developing advanced mathematical skills (Guckert, Mastropieri, & Scruggs, 2016). The "Numbers" learning area covers a large part of the Primary School (1-4th grades) Mathematics Curriculum. The main purpose of this learning area is to create a rich and robust number concept in children and to develop their processing skills.

In the first years of individuals lives, the foundations begin to form concerning the concept of numbers. Before the development of the number concept cognitively, children gain experiences related to mathematics, such as increase-decrease, and part-whole comparisons.

The experiences they have gained in this process do not include a quantitative perception and are considered a game (Griffin, 2004). Individuals have counting skills from birth. For this reason, number words begin to be used as children begin to speak (Nelson, 2007). Counting skills develop in the process from birth to primary education of individuals. In this context, it is observed that children in the age group covering this process engage in counting behaviors (Ginsburg, 2009).

The concept of number is important in terms of children's ability to make sense. As a result of a research on counting skills, it was determined that the counting skill developed in childhood affects the arithmetic skill of the individual in the later stages of his life. In this context, number counting skills form the basis for the child to be able to perform the four basic mathematical operations of addition, subtraction, multiplication and division. For this reason, the concept of counting is very important for children's lives (Stock, Desoete, & Roeyers, 2009).

The prerequisites for primary school 1st grade students to learn natural numbers are expressed in the following items (Baykul, 2016; Haylock & Manning, 2019; Olkun & Toluk Uçar, 2018)

Rhythmic Counting: At this stage, the student counts the numbers by heart but does not know the objective equivalents of the numbers. Students begin to learn the semantic equivalent of the number sequence at this stage, depending on their experiences. Although rhythmic counting develops before meaningful counting, counting skill is a cognitive activity. However, rational counting skill requires knowledge of counting principles and number words. For children to practice this, they must use their language and memory simultaneously with hand-eye coordination (Young-Loveridge, 2004). The situation that starts with rhythmic counting at the beginning continues meaningfully by matching the objects with the numbers, but still, children notice the sequence of the numbers and the order within themselves while counting rhythmically.

Regular counting (Principle of invariant order): Number words are always in the same order and counted in consecutive order. It is like when a child is asked to count six pencils, saying "one, two, three, four, five, six" and counting them in complete order. These principles are one-to-one correspondence (one and only one-word tag, e.g. "one, "two" is assigned to each counted object), stable order (the order of word tags must be invariant between counted sets). Jumping during the counting action results in an incorrect count (Geary, 2004). Similarly, Gelman and Gallistel (1986) state that children should act according to various principles to develop their counting skills. In this context, counting words should be said in order so that children can count any group of objects. They called this skill the regular-order principle. However, besides this principle, children can gain counting skills with different methods. This situation is associated with the fact that counting skills in children do not have a conceptual meaning (Olkun & Toluk Uçar, 2018). In order to count the elements in an object group, children must use a single number word for each of those objects. The fact that children can count this way shows that they have gained the one-to-one principle. In addition, the child's knowledge that the last number in the object group indicates the total number of elements indicates that the child has acquired the cardinal value principle (Olkun, Çelik, Sönmez, & Can, 2014). Although the act of counting starts in a game dimension in children, it develops in terms of symbolizing numbers and associating them with objects (Baroody, 2004). Children's association of numbers with numbers shows that they have gained experience in counting. Geary (2004) states that children in the age group of 6 can group 6 to 10 objects with the same texture by counting, associating 1 to 10 object groups with numbers, and listing numbers from 1 to 10. They also state they can use expressions such as "at least" and "at most".

One-to-One Matching: When counting a group of objects, each collection of objects must be matched with each number word. The student realizes that each object should be associated with only one number when expressing number words. Students at this stage gain the ability to count the multiplicity and find the amount sought in the multiplicity. There is a one-to-one mapping principle here. Each object is mapped to only one number word. For example, when counting 6 items, saying a number word for each item. By a one-to-one matching process between sets containing the same multiplicity, the student may notice that something is the same in the sets; in other words, multiplicities of the same amount determine an equivalence. The principles of one-to-one matching, stable order, and cardinality define counting rules, which in turn provide the skeletal framework for children's developing counting knowledge (Geary, 2004). In counting activities, one-to-one matching should be used when comparing two multiplicities.

One-to-one matching is matching sets of different objects (apples, plates, children, etc.) according to their quantity. Attention should be drawn to the difference in one-to-one matching, "How many are more?", "How many are missing?" "Is it the same?" Questions such as questions should be asked. Thus, while developing minority-many relations, and great-small relations, the preparation of the concepts of addition and subtraction is completed.

Cardinal value principle: Cardinality; the value of the last word tag represents the amount of items in the counted set (Geary, 2004). In other words, when counting objects in a group, the number said for the last counted object indicates the number of objects in the group. The last uttered number is an expression of the multiplicity counted. For example; As a result of counting 6 items, the last said number gives the number of items. So the child is no longer "How many?" However, some children think the number they say while counting one by one is the name of the last object they count. For example, when counting five beads, the child thinks five is the last bead. Therefore, it should be clear to the students that five is not the last bead but the total number of beads counted in the group. In addition to these natural constraints, children also make inferences about the basic features of counting by observing standard counting behavior and related results (Geary, 2004).

Conservation of Number (Order-independent principle): Conservation "when the physical form of any object or group of objects or its position in space changes, the object's change in number, area, volume, etc. is the principle that its properties will not change" (Senemoğlu, 2004: 43). The student understands that the number indicating the multiplicity of the object group will not change if the object group is not subjected to any processing. The child who realizes that the scattered or collective standing of the objects does not change the number of these objects performs conservation. In other words, the order of objects in counting is unimportant. For example, the result will be six counting scales/

balls in all of them, whether we count the six counting scales/balls in a row from the beginning to the end or make a circle shape and count them as a reference.

Principle of Abstraction: Counting can be applied to different groups of objects. Let the child have not only pencils but also apples, marbles, children, etc. In addition to counting, it also counts different objects in the same group. For example, like counting the apples, oranges, and bananas on the plate to determine how many fruits there are. By a one-to-one matching process between sets containing the same multiplicity, the student may notice that something is the same in the sets; in other words, multiplicities of the same amount determine an equivalence. For example, the property shared by all sets of three things is then abstracted to form the concept of 'three' as a representative of a class (cardinal number) that exists independently of any specific context. (Haylock & Manning, 2019). Initially, children construct abstract numbers based on concrete materials/objects. Then, the concept of number independent of concrete experiences is structured in children's minds.

According to Reid and Lienemann (2006), understanding numbers is "a concept that expresses the child's flexibility and fluidity in numbers, understanding what numbers mean, and the ability to perform mental mathematical operations and look around and make comparisons". These explanations lead us to the concept of number sense. In the first grade of primary school, the teaching of two-digit numbers up to 20 begins, and the teaching continues by increasing the number of digits as the grade level rises. Children gradually acquire numbers as a result of their experience because numbers are very different from concepts the child has encountered before and are abstract expressions. Since the primary school level child, who is in the concrete operational stage, creates abstract structures based on concrete objects, the numbers in the program increase as the grade level increases.

NUMBER SENSE

In the past, the successful evaluation of students in mathematics lessons depended on knowing the multiplication tables by heart and performing four operations following the rules. In contrast, in the current period, the consensus on mathematics and mathematics education is that “practice and practice” by memorizing the information does not adequately prepare students for modern life. (Anghileri, 2006). For this reason, with the educational reforms carried out, radical changes have been made in mathematics education. In the resources related to mathematics and mathematics teaching, the goals, standards, and principles of mathematics teaching that will prepare students for real life have been specified, and the concept of number sense has come to the fore among them (National Council of Teachers of Mathematics [NCTM], 2000). Number sense is developed primarily through informal and unintentional learning, that is, through the child’s daily experiences, without the need for formal learning. Number sense is a fundamental consideration for obtaining the ability to perform a proper numerical operation. For example, it is necessary for learning basic arithmetic operations. In addition, other factors play a role in the development of number processing ability. These can be personal, such as mental ability, or environmental, such as teaching at school. Likewise, sense of number is the basis of various mathematical abilities such as numbering, mental arithmetic, and estimation (Jiménez-Fernández, 2016). Although it is stated that the sense of number develops as a result of daily experiences, the effect of the learning-teaching process is undeniable.

The sense of number is not a well-known concept yet, as it is one of the newly studied subjects in our country. There are many different definitions in the literature on this subject. Some of these definitions are as follows: Howden (1989) defines number sense as “to be able to make logical predictions about different uses of numbers, to be able to notice arithmetic errors, to choose the most effective way of calculation, to recognize number patterns and to create a successful heuristic about number relations. McIntosh, Reys, and Reys (1992) and Reys, Reys, McIntosh, Emanuelsson, Johansson, and Yang (1999) define number sense as "the ability to grasp numbers and operations in general, develop useful strategies when dealing with numbers and operations, and use flexible mathematical reasoning". A similar definition was made by Berch (2005). Berch defines number sense as "the sense of the meanings of numbers. According to Berch, number sense is awareness, intuition, recognition, knowledge, skill, ability, feeling, process, conceptual structure, and mental activities. Another definition of the sense of number is stated as the ability to make sense of all the relationships related to numbers, namely few-many, part-whole, the relationship of the number with the actual amount, and the measurement results rather than knowing the number (Olkun & Toluk Uçar, 2018). A person with a sense of numbers can use numbers flexibly to make numerical expressions more meaningful and understandable, switch between different representations of numbers given to him, and associate numbers, operations, and symbols. (Markovits & Sowder, 1994). As it can be understood from the definitions made, it is seen that there is no definite and clear statement regarding the definition of number sense. In summary, sense of number requires understanding the size/smallness of numbers, using numbers flexibly in other areas of mathematics, and understanding different representations of numbers.

With the concept of number sense gaining importance in mathematics education, different from teaching students how to perform mathematical operations, it is tried to gain skills such as how to do operations from the mind, how to examine mathematical patterns, how to predict the outcome of a problem and how to talk about emerging relationships (Anghileri, 2006). Therefore, by providing students with flexible thinking skills, students can produce different solutions without depending on mathematical rules and paper and pencil.

According to NCTM (2000), student skills with high number sense are:

- Understands very well what numbers mean,
- Develops different and versatile relationships between numbers,
- When they compare numbers, they see the difference between them,
- Knows the effect of operations on numbers,
- Develops a reference point for the measurement of events taking place around it.

The fact that students with a sense of number can manage mathematical situations more easily and flexibly reveals the value of this concept in mathematics teaching. The importance of this situation has been revealed thanks to the researches carried out. (NCTM, 2000). Yang and Wu (2010) stated why teaching and learning the concept of number sense has such an essential place in four items based on various studies in the literature. (1) Number sense; It is the way of thinking flexibly, producing creative ideas and thinking logically. (2) Number sense is a holistic concept of mathematical operations and the relationship between numbers that can be used flexibly in daily life. (3) People's ability to think mathematically partially depends on their sense of number. (4) Adherence to the rules in mathematics teaching both prevents students from thinking mathematically and causes their sense of numbers not to develop. When the definitions related to the sense of numbers are examined, it can be concluded that this concept is included in a mathematical structure and an essential part of our daily life.

According to Piaget, children are in the concrete operational stage during primary school years. Children in this period may have developed the conservation principle, logical thinking, and ability to express concepts with symbols, but solving abstract problems and explaining their meanings have not yet developed despite being able to use concepts appropriately. According to Piaget, physical information about shape, color, size, and texture forms the basis of logical-mathematical knowledge. Logical-mathematical knowledge is effective in the formation of the foundations of mathematical thinking. In order to achieve this, activities such as classification, comparison, matching, and sequencing should be organized by teachers in order to develop children's mathematical skills (Ünal, 2021). Especially for children who could not reach pre-school education, such studies in the first grade should be considered as preparatory studies for the concept of numbers.

Description

Doing descriptive studies without going into the concept of number will enable children to recognize objects in their immediate surroundings (Charlesworth & Lind, 2010). For example, "What do you see in the garden?", "Show the line and ask what happened, etc. In identification studies according to their characteristics, "Does a tree have leaves?", "Does birds have wings?", "What has wings?". The pictures in the book can be discussed. "Identify the child in the picture." or "What do you see in the picture?" etc.

Pairing

Matching the elements of one set to the elements of another set is called matching. One-to-one matching refers to the situation in which one element corresponds to each element. However, one-to-one matching can be made if the attributes of the object are defined and recognized and the differences between the object and other objects are determined (Metin & Dağlıoğlu, 2006). It is stated that one-to-one matching, directly related to logical counting, is one of the basic components of the concept of numbers (Charlesworth & Lind, 2010). As a result of matching, matching groups are found by finding different groups. "Which group has more pens?" or "How many extra pens are there?" such work can be done.

Classification and Grouping

The ability to group objects according to their known properties is called classification. For the classification to be carried out by the children, the differences and similarities of the objects must be understood. Two different processes constitute the classification skill. These can be listed as follows (Charlesworth & Lind, 2010):

- Grouping of objects
- Sorting of objects

Classification can be performed by considering the characteristics of the objects. The following qualities are essential during the classification of objects (Güven, 2005):

- Number
- Colour
- Texture
- Size
- Amount
- Used area

Objects are primarily classified by children considering their characteristics, such as color and shape. Afterward, objects and thoughts are classified by considering multiple features (Lind, 2000; Sperry-Smith, 2001). Initially, vehicles are classified according to only one feature, as trucks and non-trucks, but later on, classification according to two features may be requested. For instance, they can be classified as both trucks and blue ones.

Piaget listed the classifications made by children between the ages of three and six as follows:

- mental classification
- Perceptual classification
- Multiple classifications
- Classification based on differences
- Self-classification

In perceptual grouping, the child perceives by seeing without performing any mental operations. In mental grouping, the child makes groups according to some properties of the object. In multiple grouping, it makes classification according to more than one object property. In grouping by understanding the differences, children make classification by understanding that objects have different properties. Likewise, in self-classification, the child is now at the highest classification level, understands the objective classification, and compares groups (Cantekinler, Çağdaş, & Albayrak, 2002). While making different classifications, children may also notice many mathematical skills that we cannot think of.

Compare

It is stated that the observation process and the comparison process are directly related. By observing the distinctive feature of the object by the children, the object in question is compared with other objects of the same type, and much information is obtained about the object (Lind, 2000). The objects' characteristics are considered when comparing the two groups or objects by the children. Some of these features can be listed as follows:

- Length
- Dimension
- Amount
- Speed
- Weight

Along with the features listed above, quantitative comparisons are also made by the children. The larger number of objects is included in the quantitative comparison. It is stated that the basis of ranking and measurement is comparison (Charlesworth & Lind, 2010). Comparison studies should first be based on intuition, and then continue with studies based on one-to-one matching. For example; like asking intuitively whether the items in one box are more or less than the items in another box and then matching them one-to-one.

Arrangement

The process of arranging objects by considering their features such as height, color, and weight is called sorting. It is stated that the sorting process expresses the highest level of comparison ability (Akman, Yükselen, & Uyanık, 2003). Experiences gained in daily life enable children to gain sequencing skills. They can sort the liquid in a glass from least to most, order their toys from heavy to light, and order straws from long to short. They can order the events according to time and order of occurrence (Güven, 2005). It is also a prerequisite for the objects to be sorted by different properties (from largest to smallest, from smallest to largest, etc.).

Ask for students to identify the objects they bring to the class by color, shape, size, etc. may be asked to classify and order according to their characteristics. From the largest to the smallest, rankings can be made such as large-small between two objects, larger-small among three objects, largest-smallest among five objects. For example; according to their size, such as big glass-small glass, big vase-small vase, big foot-small foot. In one-to-one matching, comparison studies can be done by matching equivalent groups and matching non-equivalent groups. For example; beans can be put in bottle caps. “More caps or beans?”, “How much more?” such questions can be asked (Nair & Pool, 1991).

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