CHAPTER 4

AN APPLICATION WITH AHP – TOPSIS AND FUZZY AHP – TOPSIS INTEGRATED APPROACH IN SMARTPHONE SELECTION

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INTRODUCTION

Technology in business and private life is increasingly intertwined (Lavrakas, 2007). Smartphones have emerged as a device that can connect to the internet like computers and can make calls and send messages like the first mobile phones. In a very short time, smartphones have become one of the indispensable needs of people. In this era of the digital world, mobile phones (communication and entertainment tool) play an important role in human life owing to their mobility, capabilities and portability (Singh, Avikal & Rashmi 2020). Smartphones, which seemed like a luxury when they first appeared on the market, have turned into a necessity. With increasing demand, countless mobile phone companies are offering increasingly complex options (Sterling, 2010). The smartphone industry has shifted to smartphone manufacturing for people in every user class. This resulted in the production of smartphones with different features or the same features with different quality.

Today it seems almost everyone is a smartphone user, particularly in developing countries and developed countries. While there is such a large market and demand, it is seen that the supply is also large. The biggest mobile phone manufacturers of the time, who could not keep up with the development of technology, could not cope with the competition and had to withdraw from the market. This has enabled new manufacturers to come to the fore.

While the majority of smartphones on the market use mobile software with Android operating system, there are also companies that use their own software. While the open source code of the Android operating system provides many advantages to users, privacy and the ability of malicious programs to access the de-

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vices are among the disadvantages of this situation. For many reasons like these, choosing a smartphone has become a complex problem. Both cost and non-cost factors should be considered when choosing a mobile phone service provider (Ju, 2009). In today's digital world, cell phones have now become an essential part of everyday life. Choosing the best mobile phone among the available alternatives is a complex decision that makes it a problem for customers (Singh et al., 2020).

In this study, the evaluation of criteria and alternatives for selection was made in line with the information received from eight different decision makers. Scoring from decision makers who are experts in the field of informatics and AHP and TOPSIS methods, which are multi-criteria decision making methods, were used together. It is seen that AHP and TOPSIS methods are frequently used together in the literature.

In this study, similar studies on AHP and TOPSIS were mentioned in the literature. Then, the definitions and application steps of AHP, TOPSIS, fuzzy AHP and fuzzy TOPSIS methods are shown. In the application part of the study, the eight most preferred phone brands in the market were compared according to eight different criteria and the selection was made. In practice, fuzzy versions of these methods have been made in addition to traditional AHP and TOPSIS. Thus, in the last part, the similar and different aspects of traditional and fuzzy methods are revealed.

LITERATURE REVIEW

In this part of the study, some of the studies that have been done with these four methods in the literature are mentioned. The studies carried out by whom, by what method and on the subject are listed below, in order of years:

Chu and Lin (2003) used fuzzy TOPSIS method for robot selection in their study. In this study, they specified fuzzy numbers and membership functions corresponding to linguistic terms in the evaluation of alternatives.

Ertuğrul and Karakaşoğlu (2008) mentioned in their study that facility location selection is a multi-criteria decision-making problem. They mentioned that fuzzy methods give better results, unlike traditional methods. In their study, they evaluated the selection of a facility location with both fuzzy AHP and fuzzy TOPSIS methods and showed similarities and differences.

Huang (2008) determined a new TOPSIS method for information system selection using combined entropy weights. With this method, he made an application by ordering a finite number of alternatives according to their criterion weights.

Ran, Pang and Li (2008) explained the basics of the fuzzy TOPSIS method in their study. They ended the study with an application by showing the basis and mathematical infrastructure of this method.

Hu, Wu and Cai (2009) made an application for the selection of the distribution center in their study. In their study, they used the fuzzy TOPSIS method for the distribution center selection. They explained the reason for choosing this method, claiming that fuzzy TOPSIS gives better results in dealing with uncertainty than traditional TOPSIS.

In his study, Amiri (2010) aimed to select the best project of a national Iranian oil company. By using AHP and fuzzy TOPSIS methods together in his study, he showed both the operation of the methods and emphasized the important differences in decision making with fuzzy numbers.

Thummala and Rao (2011) created a multi-criteria decision-making model by evaluating qualitative data for mobile phone selection. They have made an application that evaluates mobile phones by solving this model according to the AHP method.

Pan and Tian (2011) stated in their study that supplier selection for e-commerce is a difficult process. In their study, they made an application that uses AHP and Data Envelopment Analysis methods in an integrated way for supplier selection.

Dymova, Sevastjanov, and Tikhonenko (2013) mentioned in their study that the TOPSIS method is the most popular and accurate multi-criteria decision-making method. They suggested that there are too many computational errors in fuzzy studies made with this method and developed a new approach.

Jenab (2013) mentioned in his study the commercial and individual importance of mobile phone service providers. He stated that mobile service provider selection is a multi-criteria decision-making problem and alternatives are often evaluated with AHP. In this study, an application was made using the fuzzy AHP method for mobile service provider selection. TOPSIS method was used to compare the results with other methods.

Jayant, Gupta and Garg (2014) focused on reverse supply chain logistics in their study. They argued that this system, which flows from the consumer to the producer, is significantly effective in the profitability of the business. Therefore, they made an application to establish a decision support system on behalf of reverse supply chain logistics for the mobile phone industry by using the AHP and

TOPSIS method integrated.

Gupta (2016) mentioned the proliferation of mobile and wireless networks in his study. For this reason, he argued that choosing the right and reliable network is a multi-criteria decision-making problem. In his study, he demonstrated the network selection by applying the TOPSIS method.

Mayyas, Omar and Hayajneh (2016) discussed the problem of car body panel selection in their study. In their study, they argued that multi-criteria decision making was insufficient against qualitative definitions, and therefore fuzzy methods gave better results. For this reason, they made an application based on the fuzzy TOPSIS method in their study.

Nadaban, Dzitac and Dzitac (2016) showed the development of fuzzy TOPSIS method in their studies. In this study, the differences and applications of fuzzy TOPSIS are given.

In their study, Ecemiş and Yaykaşlı (2018) talked about the changes in hotel selection thanks to the developing web technology. They evaluated the hotels in the Antalya region by establishing a decision support system and database system strengthened by the TOPSIS method.

Peko, Gjeldum and Bilic (2018) mentioned additive manufacturing technologies in their studies. They suggested that the choice of additive manufacturing process is an important decision-making problem. In their studies, they showed differences and similarities by showing three different multi-criteria decision making methods together. These methods are AHP, fuzzy AHP and PROMETHEE methods.

Samanlioglu, Taşkaya and Gülen (2018) discussed the selection of new personnel to be recruited to the information technology department of a dairy company operating in Turkey. According to the criteria determined in this study, the alternatives are shown in order according to the fuzzy AHP method and according to the TOPSIS method.

In their study, Sirisawat and Kiatcharoenpol (2018) talked about the importance of the concept of reverse supply chain method and the difficulties in its implementation. In this study, an application has been made to weight the solutions for the correct implementation of reverse supply chain management. The application was completed by following the steps of fuzzy AHP and fuzzy TOPSIS methodologies.

Çaylak (2019) ranked the hotels operating in the Antalya region in his study.

In his study, he ranked the hotels according to the TOPSIS method, using the ratings given by the users in internet-based applications.

In their study, Dang, Wang, and Dang (2019) determined a development ranking by evaluating among Vietnamese cities with developing economies. In this study, fuzzy AHP and fuzzy TOPSIS were used together as a method, and they mentioned that these methods are extensions of AHP and TOPSIS and that it is a more advantageous method than traditional methods.

Kumar, Raja, Sanjeevi, Anbuudayasankar & Srihari (2019) made a selection according to the AHP method in models with different purposes for automotive selection. He included an application with AHP in his work.

Yadav, Pathak, and Gangwar (2019) argued in their study that material selection for engineering applications is a multi-criteria decision-making problem. In their studies, they made material selection for maritime applications by using TOPSIS and Preference Selection Ranking methods integrated.

Yazdani, Chatterjee and Montero-Simo (2019) focused on sustainable supply chain in their studies. For this reason, they used the AHP method and other auxiliary methods to examine the sustainable benefits and effects of mobile phones, which is a widely used product in the market.

Hassen, Halimi and Abualsauod (2020) hypothesized that yarn quality is a multi-criteria decision-making problem. They proposed a new method for evaluating yarn quality by using AHP and fuzzy numbers together.

Hoang and Nguyen (2020) investigated the factors affecting the quality of rural labor in Vietnam. In this context, they used the fuzzy AHP method to weight the criteria and sub-criteria of the rural workforce.

Singh et al. (2020) made a multi-criteria decision-making application by designing the mobile phone selection with the canoe model. In this study, criterion weighting with the fuzzy AHP method, which is one of the multi-criteria decision-making methods, and ranking of the alternatives with the TOPSIS method are adhered to.

Aytekin, Akgün, and Aydoğan (2021) aimed to choose mobile phones among university students who will constitute a certain sample in their study. In their study, they performed the sorting process by using AHP and fuzzy AHP methods together. They ended the study by determining the mobile phone model preferred by the students with a numerical application.

Çelik and Aydoğan (2021) aimed to rank among the hotels located in a district of Istanbul that attracts tourists. In this study, they finalized by taking the data from the internet sites and making an application according to the TOPSIS method.

METHOD

Analytical Hierarchy Process (AHP)

Multi-criteria decision making provides a set of procedures and techniques for solving complex decision making problems in a hierarchical manner. AHP is one of the most common MCDM methods used for site selection (Saaty, 1977, 1990). The basis of the Analytical Hierarchy Process (AHP) is a set of axioms that carefully limit the scope of the problem environment (Saaty, 1988). It is based on the well-defined mathematical structure of coherent matrices and the ability of their associated eigenvector to produce correct or approximate weights. The analytical hierarchy process compares criteria or alternatives against a criterion in a natural, binary mode. The analytical hierarchy process uses a basic scale of absolute numbers that has been proven in practice and validated by physical and decision problem experiments (Jayant et al., 2014).

AHP is a widely used analytical tool for choosing the solution of complex multi-criteria problems involving qualitative decisions (Saaty, 1980). One of the unique features of AHP is that it provides a powerful procedure for determining the relative importance of different attributes according to the purpose (Jayant et al., 2014). AHP can classify tangible and intangible factors in an organized manner. AHP has been applied many times to evaluate tangible/intangible assets, but it does not strictly limit the importance of qualitative aspects, as human thought cannot be projected onto a separate scale (Singh et al., 2020). This hierarchical structure is shown in Figure 1 below (Saaty, 1980).

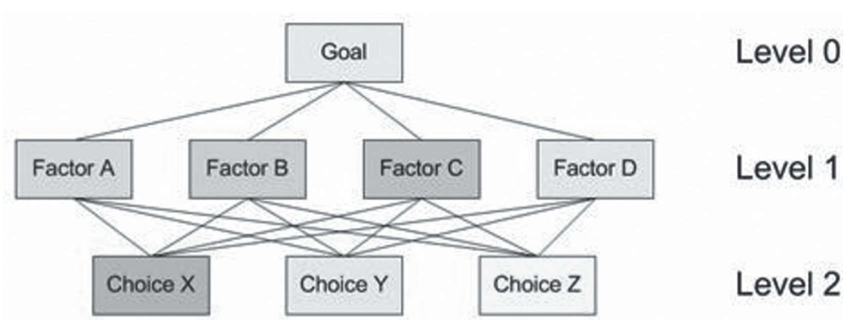

Figure 1: Representation of the AHP Hierarchy

The 9-point significance scale used by Saaty for pairwise comparisons and its explanation are shown in Table 1.

Table 1: Saaty's 1-9 point scale and explanation	
Explanation	**Importance level**
Equally Important	1
Moderately Important	3
Quite Important	5
Very important	7
Extremely Important	9
Intermediate Values	2,4,6,8

The steps of the analytical hierarchy process are given below, respectively (Saaty, 1980):

Step 1: A decision matrix is created for each decision maker with the data received from the decision makers (1).

$$Dk = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

Step 2: The geometric mean is found for each element from the k number of decision matrices obtained (2).

$$X_{ij} = \sqrt[k]{X_{ij}^{(1)} * X_{ij}^{(2)} * \ldots * X_{ij}^{(k)}} \quad (1)$$

Step 3: Based on the geometric mean result, the initial decision matrix is created (3).

$$D \quad \begin{bmatrix} x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{bmatrix} \quad (3)$$

Step 4: After matrix D created, the decision matrix normalized with (4) is created.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{m=1}^{n} x_{mj}^2}} \qquad (4)$$

Step 5: The weight values are found by taking the average of each row of the normalized decision matrix (5).

$$X_m = (X_1 + X_2 + \cdots + X_n)/n \qquad (5)$$

Step 6: Consistency testing is performed to check the consistency of the weighted values. For the consistency test, firstly, the initial decision matrix and the weight values are multiplied (6) and the result is divided by the weight values (7).

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} * \begin{bmatrix} W_1 \\ \dots \\ W_m \end{bmatrix} = \begin{matrix} Z_1 \\ \dots \\ Z_m \end{matrix} \qquad (6)$$

$$T_m = Z_m/W_m \qquad (7)$$

Step 7: Calculate ƛmax (8). The CI value is calculated (9). RI value is found from the table, RI values are shown in Table 2. The CR value is calculated (10). It is considered consistent if CR < 0.1 (Saaty, 1980).

$$ƛ_{maks} = (Z_1 + Z_2 + \cdots + Z_m)/m \qquad (8)$$

$$CI = (ƛ_{maks} - m)/m - 1 \qquad (9)$$

$$CR = CI/RI \qquad (10)$$

Table 2: RI Values

Number of Criteria	RI Value
1	0
2	0
3	0,52
4	0,89
5	1,11

6	1,25
7	1,35
8	1,40
9	1,45
10	1,49

TOPSIS

Decision problems are the process of finding the best option from all possible alternatives. In almost all multi-criteria decision problems, the variety of criteria used to evaluate alternatives is common. That is, the decision maker tries to decide on the best among the alternatives by evaluating it with many criteria (Çaylak, 2019). TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was developed by Yoon and Hwang in 1980. It can be said that the ELECTRE method adopts the basic approaches. TOPSIS constitutes the ideal solution set, in other words, by ordering the alternatives starting from the alternative that is the closest to the positive ideal solution and the farthest distance from the negative ideal solution (Ecemiş & Yaykaşlı, 2018). The TOPSIS method is generally based on the principle that the chosen alternative is the closest to the positive ideal solution and the farthest from the negative ideal solution (Çaylak, 2019).

The TOPSIS method assumes that each criterion has a monotonically increasing or decreasing utility. Therefore, positive and negative ideal solutions can be easily identified. The use of the Euclidean distance approach to evaluate the relative closeness of the alternatives to the ideal solution allows to obtain a series of comparisons of proximity distances in ordering alternative preferences (Ecemiş & Yaykaşlı, 2018). TOPSIS is widely used all over the world to rank available alternatives (Singh et al., 2020).

The steps of the TOPSIS method are given below, respectively (Hwang & Yoon, 1981):

Step 1: With the data taken from the decision makers, the decision matrix for each decision maker is created as in the equation (11).

$$D_k \quad \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \qquad (11)$$

Step 2: The geometric mean for each element from the k number of decision matrices obtained is found with the equation numbered (12).

$$X_{ij} = \sqrt[k]{X_{ij}^{(1)} * X_{ij}^{(2)} * \ldots * X_{ij}^{(k)}} \tag{12}$$

Step 3: According to the geometric mean result, the initial decision matrix is created as in the equation (13).

$$A \quad \begin{bmatrix} x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{bmatrix} \tag{13}$$

Step 4: After the decision matrix is created, the normalization matrix is created with the equation numbered (14).

$$r_{ij} = \frac{x_{ij}}{\sum_{m=1}^{n} x_{mj}^2} \tag{14}$$

(x_{ij} ; i : 1,2, …, n ; number of criteria j: 1,2, … , m; number of alternatives)

Step 5: The weighted decision matrix is created by multiplying the normalized decision matrix with the weight values as shown in the equation (15).

$$W_{mn} = w_m * N_{mn} \tag{15}$$

Step 6: In the weighted decision matrix, the positive ideal solution and the negative ideal solution are found with the equations (16) and (17).

$$A^{+} = \{(maxv_{ij} \mid j \in J), (minv_{ij} \mid j \in J')\} \tag{16}$$

$$A^{-} = \{(minv_{ij} \mid j \in J), (maxv_{ij} \mid j \in J')\} \tag{17}$$

Step 7: The distance to the positive and negative ideal solutions is found by equation (18) and (19).

$$S_j^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2} \tag{18}$$

$$S_j^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2} \tag{19}$$

Step 8: For each alternative, the relative closeness is calculated with the equation (20) and the alternatives are ranked.

$$C_i^* = \frac{S_j^-}{S_j^- + S_j^+} \qquad 0 \leq C_i^* \leq 1 \tag{20}$$

FUZZY ANALYTICAL HIERARCHY PROCESS (F-AHP)

There are many fuzzy AHP applications in the literature. Different AHP approaches were also used in these applications (Akman & Alkan, 2006). Saaty proposed the AHP method, which is the most commonly used tool for multi-criteria decision-based decisions. Considering that the AHP exhibits disadvantages due to the use of exact exact numbers (e.g. 1–9) to determine the importance of the criteria, fuzzy AHP was developed as an extension of the AHP that eliminates the disadvantages of the AHP (Dang et al., 2019). The fuzzy AHP approach is an organized approach that can be used to solve justification problems and selection of alternatives using fuzzy set theory concepts. Here, decision makers can easily categorize their choices in a pattern of ordinary languages or mathematical values regarding the importance of each selection criterion (Singh et al., 2020).

In real applications of multi-criteria decision models, it is frequently observed that decision makers verbally express their judgments or do not make objective judgments. In addition, the evaluations obtained may not always contain precise and complete information. In such decision models, analyzes can be made with fuzzy logic approach. The applications of fuzzy logic in the decision making process are generally carried out by blurring the classical decision theories (Aydın, 2016). Due to the fuzzy nature of the benchmarking process, decision makers prefer to express their pairwise comparisons on a range or verbally rather than determining them as a fixed value (Akman & Alkan, 2006).

In decision problems defined by fuzzy logic, it is aimed to reach the "best" decision that is not fuzzy as in classical problems. However, the decision obtained as a result of the fuzzy theory aims to specify the probability that each alternative can be optimal, rather than claiming the optimal decision. When there are no definite certainties in the problems; In cases where the parameters or variables are not known precisely and the evaluations are verbal, it is recommended to apply the methods developed with fuzzy theory (Aydın, 2016).

The triangular fuzzy numbers corresponding to the linguistic variables are shown in Table 3 below.

Table 3: Triangular Fuzzy Number Equivalents of Linguistic Variables

Saaty Scale	Linguistic Equivalent	Triangle Fuzzy Number
1	Equal Important	(1,1,1)
3	Weak Important	(2,3,4)
5	Quite Important	(4,5,6)
7	Strong Important	(6,7,8)
9	Absolute Important	(9,9,9)
2		(1,2,3)
4	Range Values	(3,4,5)
6		(5,6,7)
8		(7,8,9)

The steps of Fuzzy AHP are shown below in order:

Step 1: The decision matrix, which includes pairwise comparisons of the criteria, is taken as verbal values for each decision maker and is formed by transforming it into triangular fuzzy numbers as in equation (21).

$$Dk = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (21)$$

Step 2: In order to reduce more than one decision maker to a single decision matrix, the average of the values from each decision maker is found as in equation (22).

$$X_{ij} = \sqrt[k]{X_{ij}^{(1)} * X_{ij}^{(2)} * \ldots * X_{ij}^{(k)}} \quad (22)$$

Step 3: In order to weight the weights consisting of triangular fuzzy numbers on the basis of criteria, the equation in equation (23) and geometric mean are found.

$$r_i = \left(\prod_{J=1}^{n} d_{ij}\right)^{1/n}, i = 1,2, \ldots, n \tag{23}$$

Step 4: According to the averages of the decision makers, the decision matrix is updated as in equation (24).

$$\mathrm{D} = \begin{bmatrix} x_{\mathrm{lr1}} & x_{\mathrm{mr2}} & x_{\mathrm{ur3}} \\ \vdots & \ddots & \vdots \\ x_{lr1} & x_{\mathrm{mr2}} & x_{ur3} \end{bmatrix} \tag{24}$$

Step 5: The sum of the column values of the matrix formed with weighted triangular fuzzy numbers of the criteria is found with the equation (25).

$$d_l = \sum_{s=1}^{r} x_{ls} \;,\; d_m = \sum_{s=1}^{r} x_{ms} \,,\; d_u = \sum_{s=1}^{r} x_{us} \tag{25}$$

(r: number of lines)

Step 6: The average weight values of the criteria are found by the equation (26) as a result of cross-dividing the matrix (24) by the values in the equation (25).

$$\mathrm{D} = \begin{bmatrix} x_{\mathrm{lr1}}/d_u & x_{\mathrm{mr2}}/d_m & x_{\mathrm{ur3}}/d_l \\ \vdots & \ddots & \vdots \\ x_{lr1}/d_u & x_{\mathrm{mr2}}/d_m & x_{ur3}/d_l \end{bmatrix} \tag{26}$$

In fuzzy AHP, after the criteria are weighted, each alternative is evaluated separately for all criteria. The steps after this stage are the steps applied only when fuzzy AHP is used in this study. As a result of multiplying each alternative with the criterion weights, the weighted triangular fuzzy number values of the alternatives are found. Triangular fuzzy number values are found as lower value (l), middle value (m) and upper value (u) for each alternative and summed to find the total weighted triangular fuzzy number values. Afterwards, the following steps are followed one by one for each alternative:

Step 7: The lower limit value (LB) of the alternatives is found by the equation (27). The value of an expressed in this equation consists of rational numbers starting from 0.1 to 0.9. There are 9 LB values in total.

$$LB = \{[(m - l) * a_n] + l\} \tag{27}$$

Step 8: The upper limit value (UB) of the alternatives is found by the equation (28). The value of an expressed in this equation consists of rational numbers starting from 0.1 to 0.9. There are 9 UB values in total.

$$UB = \{u - [(u - m) * a_n]\} \tag{28}$$

Step 9: Average LB and Average UB values for each alternative are found by equation (29) and (30).

$$\textstyle\sum_1^9 (a_n * LB_n) \tag{29}$$

$$\textstyle\sum_1^9 (a_n * UB_n) \tag{30}$$

Step 10: Lower bound and upper bound weight values for each alternative are found by equation (31) and (32).

$$W_{A_i(LB)} = \left(\textstyle\sum_1^9 (a_n * LB_n)/4{,}5\right) \tag{31}$$

$$W_{A_i(UB)} = \left(\textstyle\sum_1^9 (a_n * UB_n)/4{,}5\right) \tag{32}$$

Step 11: The fuzzy weight values for each alternative are clarified with the equation (33).

$$W_{dA_i} = \left[\delta * W_{A_i(UB)} + (1 - \delta) * W_{A_i(LB)}\right] \tag{33}$$

Step 12: The pure weights of the alternatives are normalized with the equation (34) and sorted.

$$W_{n(A_i)} = \left(\frac{W_{dA_i}}{\sum_i^j W_{dA_i}}\right) \tag{34}$$

FUZZY TOPSİS (F-TOPSİS)

TOPSIS method is a multi-criteria decision making problem with m alternatives as a geometric system with m points in n-dimensional space. Based on the concept of alternative choice, this method has the closest distance to the positive – ideal solution and the farthest distance to the negative – ideal solution. In the TOPSIS method, an index called positive – similarity to ideal solution and distance to negative – ideal solution is defined. With this definition, the method chooses an alternative with maximum similarity to the ideal solution. There are some fuzzy TOPSIS methods developed in the literature. The differences between these methods are due to the computational techniques. While some authors used triangular fuzzy numbers, others used trapezoidal fuzzy numbers (Özdemir & Seç, 2009). The fuzzy TOPSIS method is a method that helps to evaluate and rank the alternatives under uncertainty according to a certain criterion or criteria and to make the most accurate choice (Çınar, 2010).

Fuzzy TOPSIS is often used to rank different alternatives based on the shortest and farthest distances from the positive-ideal solution and the negative-ideal solution, respectively (Dang et al., 2019). The use of fuzzy values in the TOPSIS method started in 1992 with Chen and Hwang's reference to the study on the classical TOPSIS method. After that, this method has been used to solve many multi-criteria decision making problems. In the fuzzy TOPSIS method proposed by Chen, the determination of criterion weights and the evaluation of alternatives are made with verbal variables expressed with triangular fuzzy numbers (Çınar, 2010).

The steps of Fuzzy TOPSIS are shown below in order:

Step 1: Verbal rating values for the alternatives are taken from each decision maker and converted to triangular fuzzy numbers as in the equation (35).

$$D_k \qquad \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \qquad (35)$$

Step 2: The geometric mean of triangular fuzzy numbers for each element from the k number of decision matrices obtained is found with the equation (36).

$$X_{ij} = \sqrt[k]{X_{ij}^{(1)} * X_{ij}^{(2)} * \ldots * X_{ij}^{(k)}} \qquad (36)$$

Step 3: According to the geometric mean result, the beginning decision matrix is shown as in the equation (37).

$$D \quad \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (37)$$

Step 4: After the matrix D is created, the normalization matrix is created with (38).

$$r_{ij} = \frac{x_{ij}}{\sum_{m=1}^{n} x_{mj}^2} \quad (38)$$

(x_{ij} ; i : 1,2, …, n ; number of criteria j: 1,2, … , m; number of alternatives)

Step 5: The weighted decision matrix is created by multiplying the normalized decision matrix with the weight values as in the equation (39).

$$W_{mn} = w_m * N_{mn} \quad (39)$$

Step 6: With the equations (40) and (41) in the weighted decision matrix, the positive ideal solution and the negative ideal solution are found separately for each criterion.

$$A^+ = \{(maxv_{ij} \mid j \in J),\ (minv_{ij} \mid j \in J')\} \quad (40)$$

$$A^- = \{(minv_{ij} \mid j \in J),\ (maxv_{ij} \mid j \in J')\} \quad (41)$$

Step 7: The clarified values in the weighted decision matrix are calculated for each row by adding the squares of the difference from the positive ideal solution and the negative ideal solution calculated in its own column, and the square root of the mean is found with the equation (42) and (43).

$$D^+ = \sqrt{\left[\frac{1}{3}\left(\left(\sum_{j=1}^{3}(v_{ij} - A^+)^2\right)\right)\right]} \quad (42)$$

$$D^- = \sqrt{\left[\frac{1}{3}\left(\left(\sum_{j=1}^{3}\left(v_{ij} - A^-\right)^2\right)\right)\right]} \tag{43}$$

Step 8: The distance to the positive and negative ideal solutions is found separately for each alternative with the equation (44) and (45).

$$S_j^+ = \sum_{j=1}^{n}(D_{ij}^+) \tag{44}$$

$$S_j^- = \sum_{j=1}^{n}(D_{ij}^-) \tag{45}$$

(n: number of columns)

Step 9: For each alternative, the relative closeness is calculated with the equation (46) and the alternatives are ranked.

$$C_i^* = \frac{S_j^-}{S_j^- + S_j^+} \qquad 0 \leq C_i^* \leq 1 \tag{46}$$

APPLICATION

In this study, two different decision making methods were used together for the decision making problem. In the weighting of the criteria, the AHP method, which has been demonstrated by previous studies, was used in the selection of the alternatives, and the TOPSIS method, which was successful in reaching the right result with its geometric infrastructure. The integrated use of these two methods is frequently encountered in the literature. However, in this study, the integrated approach of these two methods and the integrated approach in fuzzy structure are discussed together. In this way, the similarities and differences between the results of the traditional method and the fuzzy method have been revealed.

SCOPE AND PURPOSE OF THIS STUDY

The aim of this study is to choose among 8 alternatives in line with eight different criteria among the top or top class member devices released by smartphone manufacturers. In the evaluation of each criterion and alternative, eight decision makers who are experts in the field of informatics were consulted. Decision makers independently evaluated each criterion within itself and then each alternative within the scope of the criterion, and numerical results were obtained by using multi-criteria decision making methods. Decision makers evaluated each criterion on the basis of criteria/performance while evaluating the criteria. For example; price/performance, camera resolution/performance. In the application part, first, the criteria according to the traditional method were evaluated with AHP and the alternatives were evaluated with TOPSIS and a result was reached. Then, according to the fuzzy method, the criteria were evaluated with BAHP and the alternatives were evaluated with BTOPSİS and a new result was reached. Within the scope of the study, both results were compared with each other, similarities and differences and their reasons were discussed. The criteria used in this study are shown in Table 4.

Table 4: Criteria

CRITERIA	EXPLANATION
Price (K1)	Market value of smartphone (₺)
Prestige (K2)	The image of the smartphone in society
Camera Resolution (K3)	Resolution of the rear cameras of the smartphone (MP)
Battery Capacity (K4)	Battery capacity of the smartphone (mAh)
RAM Capacity (K5)	Temporary memory capacity of the smartphone (GB)
Internal Storage (K6)	Fixed storage of the smartphone (GB)
Screen Size (K7)	Smartphone screen size (inch)
Weight (K8)	Total weight of the smartphone (g)

The features of 8 smartphone models determined as an alternative to the application within the scope of this study, according to the above criteria, are shown in Table 3. Since Prestige is a relative concept among these features, it was determined by taking the personal opinion of the decision makers. Therefore, it is not shown in Table 5.

Table 5: Alternatives

ALTERNATIVES	C1	C3	C4	C5	C6	C7	C8
Xiaomi Redmi Note 11 Pro	8280	118	5000	8	128	6,67	202
Oppo Reno 6	7374	76	4310	8	128	6,40	175
Reeder P13 Blue Max Pro	3781	56	4000	8	128	6,55	150
Apple Iphone 13	19608	24	3095	4	256	6,10	173
Casper Via X20	3919	60	4510	6	128	6,53	190
Samsung Galaxy S22 Ultra	23667	140	5000	12	256	6,80	228
General Mobile GM22 Pro	4515	127	5000	8	128	6,70	213

Huawei P40 Pro	14601	68	4200	8	256	6,55	209

The characteristics of the alternatives were filled with the information obtained from the website of Hepsiburada.com and presented to the decision makers.

DATA COLLECTION

The fixed data (price, camera resolution, battery capacity, etc.) obtained in this study were obtained from the website Hepsiburda.com. Alternatives to be used in the application were created by taking the flagship of certain brands or a device in this class. For the decision-making process, scoring was made according to the information obtained from 8 decision makers who are experts in the field of informatics and have a command of the smartphone industry. Decision makers made the scoring according to Saaty's 1-9 scale. Then, in the fuzzy part of the application, these values were converted to the corresponding triangular fuzzy numbers.

EVALUATION OF RESEARCH FINDINGS

Traditional AHP-TOPSIS Integrated Approach

Step 1: AHP management was used to compare and evaluate the criteria with each other. The criteria chosen for the comparison of the alternatives are defined by coding so that they can be easily demonstrated in practice; C1: Price, C2: Prestige, : Camera Resolution, C4: Battery Capacity, C5: RAM Capacity, 6: Internal Storage, C7: Screen Size, C8: Weight. Decision makers who are experts in the field of informatics evaluated the criteria on Saaty's 1-9 scale and are shown as in the equation (1). Then, the geometric mean of the pairwise comparison value of each decision maker was taken with the equation (2), and Table 6 was formed as in the equation (3).

Table 6: Comparison of Criteria (AHP)

	C1	C2	C3	C4	C5	C6	C7	C8
C1	1,000	3,035	4,107	2,611	3,609	3,797	4,907	6,370
C2	0,329	1,000	1,939	0,951	1,351	1,223	3,219	4,318
C3	0,243	0,516	1,000	0,570	0,799	0,859	2,956	4,733
C4	0,383	1,052	1,755	1,000	2,449	2,243	5,354	6,226
C5	0,277	0,740	1,251	0,408	1,000	1,000	2,729	4,015
C6	0,263	0,818	1,164	0,446	1,000	1,000	3,816	5,081

C7	0,204	0,311	0,338	0,187	0,366	0,262	1,000	2,246
C8	0,157	0,232	0,211	0,161	0,249	0,197	0,445	1,000

Step 2: The decision matrix used in the criterion comparison was normalized with the equation (4), and the normalized criterion comparisons shown in Table 7 were obtained.

Table 7: Normalized Criteria Comparisons

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
C1	0,350	0,394	0,349	0,412	0,333	0,359	0,201	0,187
C2	0,115	0,130	0,165	0,150	0,125	0,116	0,132	0,127
C3	0,085	0,067	0,085	0,090	0,074	0,081	0,121	0,139
C4	0,134	0,137	0,149	0,158	0,226	0,212	0,219	0,183
C5	0,097	0,096	0,106	0,064	0,092	0,095	0,112	0,118
C6	0,092	0,106	0,099	0,070	0,092	0,095	0,156	0,149
C7	0,071	0,040	0,029	0,029	0,034	0,025	0,041	0,066
C8	0,055	0,030	0,018	0,025	0,023	0,019	0,018	0,029

Step 3: The weight values of the row averages of the normalized criterion comparisons are found in the equation (5). The weights of the criteria are shown in Table 8.

Table 8: Weight Values of Criteria

Criteria	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Weights	0,323	0,132	0,093	0,177	0,098	0,108	0,042	0,027

Step 4: To test the consistency of the model after weighting, the λ value is found by the equation (8) and the CI value is found by the equation (9). Since this model includes 8 criteria, the RI value was found from the table and taken as 1.40. The CR value of the model is consistent as it is $0.024 < 0.1$ according to the calculations made as in the equation (10). The parameters of the consistency test of the model are shown in Table 9.

Table 9: λ, CI, RI and CR Values			
λ	CI	RI	CR
8,242	0,035	1,410	0,024

Step 5: TOPSIS method is used to evaluate the alternatives. In order to create the decision matrix, the information obtained from the decision makers who are experts in the field of informatics and the data of Hepsiburada.com were used. Alternatives are defined by coding in order to show the comparison of alternatives easily in practice; A1: Xiaomi Redmi Note 11 Pro, A2: Samsung Galaxy S22 Ultra, A3: Apple Iphone 13, A4: Casper Via X20, A5: General Mobile GM22 Pro, A6: Reeder P13 Blue Max Pro, A7: Huawei P40 Pro, A8: Oppo Reno 6. Each decision maker evaluated each alternative for all criteria as in the equation numbered (11) on Saaty's scale of 1-9, and the geometric mean of these values was taken as in the equation numbered (12), and Table 10 was formed as in the equation numbered (13).

Table 10: Decision Matrix (TOPSIS)								
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
A1	4,449	7,263	7,560	8,469	6,718	5,281	7,205	5,354
A2	3,386	3,327	4,072	4,867	6,718	5,281	4,292	7,707
A3	4,789	1,000	1,542	3,591	6,718	5,281	5,979	8,739
A4	6,481	9,000	8,868	2,847	4,174	7,430	2,747	8,223
A5	4,843	1,542	3,330	6,318	4,681	5,281	5,555	6,590
A6	3,064	8,186	8,469	8,469	8,594	8,594	8,485	1,622
A7	2,810	2,482	7,190	8,469	6,718	5,281	7,594	3,060
A8	2,135	5,871	4,441	4,932	6,718	5,281	6,095	4,196

Step 6: Normalized decision matrix is created. After the decision matrix was created, the matrix was normalized with the equation (14). The normalized decision matrix is shown in Table 11.

Table 11: Normalized Decision Matrix (TOPSIS)								
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
A1	0,374	0,454	0,430	0,470	0,365	0,307	0,606	0,335
A2	0,285	0,208	0,232	0,270	0,365	0,307	0,361	0,482
A3	0,403	0,063	0,088	0,199	0,365	0,307	0,503	0,546

A4	0,545	0,563	0,505	0,158	0,227	0,432	0,231	0,514
A5	0,407	0,096	0,189	0,351	0,254	0,307	0,467	0,412
A6	0,258	0,512	0,482	0,470	0,467	0,499	0,713	0,101
A7	0,236	0,155	0,409	0,470	0,365	0,307	0,638	0,191
A8	0,180	0,367	0,253	0,274	0,365	0,307	0,512	0,262

Step 7: The weights determined in Table 7 for the criteria are multiplied by each element of the normalized decision matrix shown in Table 10, as in the equation (15), and a weighted decision matrix is formed. The weighted decision matrix is given in Table 12.

Table 12: Weighted Decision Matrix (TOPSIS)

	C1	C2	C3	C4	C5	C6	C7	C8
A1	0,121	0,060	0,057	0,083	0,036	0,033	0,196	0,044
A2	0,092	0,028	0,031	0,048	0,036	0,033	0,117	0,064
A3	0,130	0,008	0,012	0,035	0,036	0,033	0,162	0,072
A4	0,176	0,075	0,067	0,028	0,022	0,046	0,075	0,068
A5	0,132	0,013	0,025	0,062	0,025	0,033	0,151	0,055
A6	0,083	0,068	0,064	0,083	0,046	0,054	0,231	0,013
A7	0,076	0,021	0,054	0,083	0,036	0,033	0,206	0,025
A8	0,058	0,049	0,033	0,049	0,036	0,033	0,166	0,035

Step 8: Positive ideal (A+) is found by equation (16) and negative ideal (A-) is found by equation (17). Positive ideal solutions represent the best performance values, and negative ideal solutions represent the worst performance results. Positive and negative ideal solutions are shown in Table 13.

Table 13: Positive and Negative Ideal Solutions (TOPSIS)

	C1	C2	C3	C4	C5	C6	C7	C8
(A+)	0,176	0,075	0,067	0,083	0,046	0,054	0,231	0,072
(A-)	0,058	0,008	0,012	0,028	0,022	0,033	0,075	0,013

Step 9: The distance of the weighted values from the positive ideal solutions (S+) was calculated by the equation (18) and the distance from the negative ideal solutions (S-) by the equation (19). The results are shown in Table 14.

Table 14: Discrimination Measurements (TOPSIS)		
	S+	S-
A1	0,072	0,180
A2	0,127	0,095
A3	0,112	0,143
A4	0,118	0,159
A5	0,099	0,133
A6	0,124	0,202
A7	0,130	0,167
A8	0,138	0,122

Step 10: The closeness to the ideal solution (C*) was calculated according to the formula (20) and sorted. The ranking is shown in Table 15.

Table 15: Ranking of Alternatives (TOPSIS)		
Ranking	**Smartphone**	**Result (C*)**
1	Xiaomi Redmi Note 11 Pro	0,714
2	Samsung Galaxy S22 Ultra	0,619
3	Apple Iphone 13	0,574
4	Casper Via X20	0,572

Table 16: Comparison of Criteria (FAHP)								
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
	1,000	0,262	0,195	0,274	0,215	0,208	0,167	0,137
C1	1,000	0,329	0,243	0,383	0,277	0,263	0,204	0,157
	1,000	0,462	0,327	0,648	0,394	0,361	0,266	0,157
	2,163	1,000	0,397	0,837	0,529	0,599	0,233	0,185
C2	3,035	1,000	0,516	1,052	0,740	0,818	0,311	0,232
	3,820	1,000	0,707	1,351	1,091	1,195	0,479	0,316
	3,060	1,414	1,000	1,233	0,892	0,879	0,270	0,174
C3	4,107	1,939	1,000	1,755	1,251	1,164	0,338	0,211
	5,136	2,519	1,000	2,482	1,778	1,542	0,462	0,271
	1,542	0,740	0,403	1,000	0,311	0,329	0,157	0,138
C4	2,611	0,951	0,570	1,000	0,408	0,446	0,187	0,170
	3,644	1,195	0,811	1,000	0,616	0,733	0,230	0,193

5	General Mobile GM22 Pro	0,562
6	Reeder P13 Blue Max Pro	0,561
7	Huawei P40 Pro	0,469
8	Oppo Reno 6	0,428

Fuzzy AHP-TOPSIS Integrated Approach

Step 1: Fuzzy AHP management was used to compare and evaluate the criteria with each other. The criteria chosen for the comparison of the alternatives are defined by coding so that they can be easily demonstrated in practice; C1: Price, C2: Prestige, C3: Camera Resolution, C4: Battery Capacity, C5: RAM Capacity, C6: Internal Storage, C7: Screen Size, C8: Weight. Expert decision makers in the field of informatics evaluated the criteria on Saaty's 1-9 scale. Evaluations were converted into triangular fuzzy numbers as shown in equation (21). Lower (l), middle (m) and upper (u) limit values were found for each decision maker. Then, the geometric mean of the pairwise comparison value of each decision maker was calculated using the equation (22) and Table 16 was formed.

Table 16: Comparison of Criteria (continued) (FAHP)

	C5			C6			C7			C8		
C1	2,539	3,609	4,643	2,769	3,797	4,813	3,764	4,907	5,979	5,392	6,370	7,311
C2	0,917	1,351	1,889	0,837	1,223	1,668	2,087	3,219	4,284	3,165	4,318	5,403
C3	0,562	0,799	1,121	0,648	0,859	1,138	2,163	2,956	3,709	3,693	4,733	5,759
C4	1,622	2,449	3,219	1,364	2,243	3,040	4,349	5,354	6,357	5,193	6,226	7,248
C5	1,000	1,000	1,000	0,872	1,091	1,223	1,646	2,729	3,766	2,908	4,015	5,068
C6	0,818	1,000	1,147	1,000	1,000	1,000	2,729	3,816	4,860	3,994	5,081	6,135
C7	0,260	0,356	0,586	0,206	0,262	0,366	1,000	1,000	1,000	1,414	2,246	3,219
C8	0,194	0,244	0,336	0,163	0,197	0,250	0,311	0,445	0,707	1,000	1,000	1,000

Step 2: For the decision matrix used in the criterion comparison, the geometric mean of each lower (l), middle (m) and upper (u) limit values on a row basis is taken as shown in the equation (23) and the new matrix is formed as in the equation (24). The triangular fuzzy numbers shown in Table 17 were formed and the mean matrix was obtained. The total values of each column were calculated at the end of the column as shown in the equation (25).

Table 17: Mean Boundary Values of Triangular Fuzzy Numbers			
	Lower Boundary (l)	**Middle Limit (m)**	**Upper Boundary (u)**
C1	2,486	3,296	4,032
C2	1,042	1,390	1,782
C3	0,741	0,954	1,226
C4	1,393	1,837	2,366
C5	0,776	1,028	1,349
C6	0,868	1,102	1,422
C7	0,325	0,413	0,558
C8	0,222	0,270	0,341
TOTAL	7,852	10,290	13,075

Step 3: The triangular limit values of the criteria were weighted with the equation (26) and the criteria weights were calculated. The weights of the criteria are shown in Table 18.

Table 18: Triangular Fuzzy Number Weights of Criteria			
	Lower Boundary (l)	**Middle Limit (m)**	**Upper Boundary (u)**
C1	0,190	0,320	0,513
C2	0,080	0,135	0,227
C3	0,057	0,093	0,156
C4	0,107	0,178	0,301
C5	0,059	0,100	0,172
C6	0,066	0,107	0,181
C7	0,025	0,040	0,071
C8	0,017	0,026	0,043

Step 4: The fuzzy TOPSIS method is used to appraise the alternatives. In order to create the decision matrix, the information obtained from the decision makers who are experts in the field of informatics and the data of Hepsiburada.com were used. Alternatives are defined by coding in order to show the comparison of alternatives easily in practice; A1: Xiaomi Redmi Note 11 Pro, A2: Samsung Galaxy S22 Ultra, A3: Apple Iphone 13, A4: Casper Via X20, A5: General Mobile GM22 Pro, A6: Reeder P13 Blue Max Pro, A7: Huawei P40 Pro, A8: Oppo Reno 6. Each decision maker evaluated each alternative for all criteria on Saaty's 1-9 scale, and these values were converted to triangular fuzzy numbers as in the equation (35).

The geometric mean of these values was found as in the equation (36) and Table 19 was formed with the equation (37).

Table 19: Decision Matrix (FTOPSIS)												
	C1			**C2**			**C3**			**C4**		
A1	3,436	4,449	5,458	6,335	7,263	8,169	6,877	7,687	8,469	8,034	8,469	8,868
A2	2,328	3,386	4,414	2,502	3,327	4,086	2,885	4,072	5,167	3,828	4,867	5,892
A3	3,766	4,789	5,803	1,000	1,000	1,000	1,000	1,542	1,987	2,519	3,591	4,627
A4	5,477	6,481	7,483	9,000	9,000	9,000	8,722	8,868	9,000	2,104	3,193	4,236
A5	3,797	4,843	5,871	1,000	1,542	1,987	2,482	3,330	4,105	5,308	6,318	7,326
A6	2,030	3,064	4,080	7,610	8,186	8,722	8,034	8,469	8,868	8,034	8,469	8,868
A7	1,769	2,810	3,828	1,414	2,482	3,515	6,435	7,190	7,896	8,034	8,469	8,868
A8	1,707	2,135	2,515	4,843	5,871	6,890	3,386	4,441	5,474	3,916	4,932	5,943

Table 19: Decision Matrix (continued) (FTOPSIS)												
	C5			C6			C7			C8		
A1	5,713	6,718	7,722	4,258	5,281	6,296	6,290	7,205	8,103	4,349	5,354	6,357
A2	5,713	6,718	7,722	4,258	5,281	6,296	3,266	4,292	5,308	6,901	7,707	8,485
A3	5,713	6,718	7,722	4,258	5,281	6,296	4,975	5,979	6,982	8,452	8,739	9,000
A4	3,151	4,174	5,188	6,514	7,430	8,328	1,929	2,747	3,510	7,545	8,223	8,868
A5	3,663	4,681	5,692	4,258	5,281	6,296	4,538	5,555	6,567	5,584	6,590	7,594
A6	8,291	8,594	8,868	8,291	8,594	8,868	7,937	8,485	9,000	1,091	1,622	2,060
A7	5,713	6,718	7,722	4,258	5,281	6,296	6,688	7,594	8,485	1,958	3,060	4,107
A8	5,713	6,718	7,722	4,258	5,281	6,296	5,089	6,095	7,099	3,177	4,196	5,207

Step 5: Normalized decision matrix is created. After the decision matrix was created, the matrix was normalized with the formula (38). The normalized decision matrix is shown in Table 20.

Table 20: Normalized Decision Matrix (FTOPSIS)												
	C1			**C2**			**C3**			**C4**		
A1	0,237	0,374	0,589	0,363	0,454	0,560	0,357	0,436	0,529	0,402	0,469	0,547
A2	0,160	0,285	0,476	0,143	0,208	0,280	0,150	0,231	0,323	0,192	0,270	0,364
A3	0,259	0,403	0,626	0,057	0,063	0,069	0,052	0,087	0,124	0,126	0,199	0,286

A4	0,377	0,545	0,807	0,515	0,563	0,617	0,453	0,503	0,562	0,105	0,177	0,261
A5	0,261	0,407	0,633	0,057	0,096	0,136	0,129	0,189	0,256	0,266	0,350	0,452
A6	0,140	0,258	0,440	0,436	0,512	0,598	0,417	0,480	0,554	0,402	0,469	0,547
A7	0,122	0,236	0,413	0,081	0,155	0,241	0,334	0,408	0,493	0,402	0,469	0,547
A8	0,117	0,180	0,271	0,277	0,367	0,472	0,176	0,252	0,342	0,196	0,273	0,367

Table 20: Normalized Decision Matrix (continued) (FTOPSIS)

	C5			**C6**			**C7**			**C8**		
A1	0,274	0,365	0,483	0,217	0,307	0,425	0,314	0,408	0,531	0,224	0,307	0,409
A2	0,274	0,365	0,483	0,217	0,307	0,425	0,163	0,243	0,348	0,355	0,441	0,546
A3	0,274	0,365	0,483	0,217	0,307	0,425	0,248	0,339	0,457	0,435	0,500	0,580
A4	0,151	0,227	0,325	0,332	0,432	0,562	0,096	0,156	0,230	0,388	0,471	0,571
A5	0,175	0,254	0,356	0,217	0,307	0,425	0,227	0,315	0,430	0,287	0,377	0,489
A6	0,397	0,467	0,555	0,422	0,499	0,598	0,396	0,481	0,590	0,056	0,093	0,133
A7	0,274	0,365	0,483	0,217	0,307	0,425	0,334	0,430	0,556	0,101	0,175	0,264
A8	0,274	0,365	0,483	0,217	0,307	0,425	0,254	0,345	0,465	0,163	0,240	0,335

Step 6: Weighted decision matrix is created. The weights determined in Table 17 for the criteria were multiplied by each element of the normalized decision matrix in Table 19 as in the equation (39), and a weighted decision matrix was formed. The weighted decision matrix is given in Table 21.

Table 21: Weighted Decision Matrix (FTOPSIS)

	C1			**C2**			**C3**			**C4**		
A1	0,045	0,120	0,302	0,029	0,061	0,127	0,020	0,040	0,083	0,043	0,084	0,165
A2	0,030	0,091	0,244	0,011	0,028	0,064	0,008	0,021	0,050	0,020	0,048	0,110
A3	0,049	0,129	0,321	0,005	0,008	0,016	0,003	0,008	0,019	0,013	0,035	0,086
A4	0,072	0,175	0,414	0,041	0,076	0,140	0,026	0,047	0,088	0,011	0,032	0,079
A5	0,050	0,130	0,325	0,005	0,013	0,031	0,007	0,018	0,040	0,028	0,062	0,136
A6	0,027	0,083	0,226	0,035	0,069	0,136	0,024	0,045	0,087	0,043	0,084	0,165
A7	0,023	0,076	0,212	0,006	0,021	0,055	0,019	0,038	0,077	0,043	0,084	0,165
A8	0,022	0,058	0,139	0,022	0,050	0,107	0,010	0,023	0,053	0,021	0,049	0,110
	C5			**C6**			**C7**			**C8**		

A1	0,016	0,036	0,083	0,014	0,033	0,077	0,008	0,016	0,038	0,004	0,008	0,018
A2	0,016	0,036	0,083	0,014	0,033	0,077	0,004	0,010	0,025	0,006	0,012	0,024
A3	0,016	0,036	0,083	0,014	0,033	0,077	0,006	0,014	0,032	0,007	0,013	0,025
A4	0,009	0,023	0,056	0,022	0,046	0,102	0,002	0,006	0,016	0,007	0,012	0,025
A5	0,010	0,025	0,061	0,014	0,033	0,077	0,006	0,013	0,031	0,005	0,010	0,021
A6	0,024	0,047	0,095	0,028	0,053	0,108	0,010	0,019	0,042	0,001	0,002	0,006
A7	0,016	0,036	0,083	0,014	0,033	0,077	0,008	0,017	0,039	0,002	0,005	0,011
A8	0,016	0,036	0,083	0,014	0,033	0,077	0,006	0,014	0,033	0,003	0,006	0,015

Step 7: Positive ideal (A+) and negative ideal (A-) solutions are found. Positive ideal solutions represent the best performance values, and negative ideal solutions represent the worst performance results. The positive ideal solution is calculated with the equation (40) and the negative ideal solution is calculated with the equation (41) and shown in Table 22.

Table 22: Positive and Negative Ideal Solutions (FTOPSIS)

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
(A+)	0,414	0,140	0,088	0,165	0,095	0,108	0,042	0,025
(A-)	0,022	0,005	0,003	0,011	0,009	0,014	0,002	0,001

Step 8: The weighted values are clarified with the equations (42) and (43) in order to calculate the distance from the positive ideal solutions (S+) and the distance from the negative ideal solutions (S-). (S+) and (S-), (44) and (45) numbered equations. The results are shown in Table 23.

Table 23: Discrimination Measurements (FTOPSIS)

	S+	S-
A1	0,662	0,519
A2	0,762	0,373
A3	0,774	0,364
A4	0,658	0,539
A5	0,745	0,404
A6	0,668	0,506
A7	0,742	0,405
A8	0,779	0,340

Step 9: The closeness to the ideal solution (C*) was calculated according to the equation (46) and sorted. The ranking is shown in Table 24.

Table 24: Ranking of Alternatives (FTOPSİS)

Ranking	Smartphone	Results (C*)
1	Apple Iphone 13	0,450
2	Xiaomi Redmi Note 11 Pro	0,439
3	Samsung Galaxy S22 Ultra	0,431
4	General Mobile GM22 Pro	0,353
5	Casper Via X20	0,352
6	Oppo Reno 6	0,329
7	Reeder P13 Blue Max Pro	0,320
8	Huawei P40 Pro	0,304

CONCLUSION

In this study, the significance of smartphones, which are the main players in the significance of fast communication in recent years, has been emphasized. As smartphones gain importance for individuals and businesses, competition has arisen for companies that make money from this industry. In fact, even the world giants who could not keep up with this technological progress had to withdraw from the market by sinking. Although different companies have taken over the global market over time, models suitable for every type of need and budget have begun to appeal to different segments. For this reason, instead of producing devices with certain features, smartphone manufacturers have created a production standard that can appeal to all kinds of users.

In this study, the issue of smartphone selection is discussed. While establishing the model, the solution of the issue was approached with the AHP and TOPSIS methods, which are the most popular among the multi-criteria decision making methods. In this study, the AHP and TOPSIS method were used together, that is, integrated, as in many studies in the literature. In the literature, it is a common situation in the studies examined that the use of these methods together yields more realistic results. However, the different feature of this study is that it uses the fuzzy AHP-TOPSIS integrated approach together with the traditional AHP-TOPSIS integrated approach. Thus, it has become possible to reveal the similar and different aspects of both approaches.

In this study, the observation, knowledge and experience of eight decision makers who are experts in the field of informatics were consulted for the selection of smartphones. Decision makers have used a qualitative judgment-based scoring method based not only on their own thoughts, but also on the values given to the criteria and alternatives by the smartphone users around them. In this way, a sample judgment has emerged that can represent the society. The data from the decision makers were first evaluated according to the traditional AHP-TOPSIS integrated approach and then evaluated according to the fuzzy AHP-TOPSIS integrated approach, which was renewed with fuzzy numbers. In this way, it was desired to determine whether there is a difference between the two methods and, if so, how large this difference is.

8 criteria by expert decision makers for implementation; Price is determined by Prestige, Camera Resolution, Battery Capacity, RAM Capacity, Internal Storage, Screen Size and Weight. These criteria are weighted according to the AHP and fuzzy AHP methods. Then, 8 different alternative smartphone models for these criteria; It is designated as Xiaomi Redmi Note 11 Pro, Samsung Galaxy S22 Ultra, Apple Iphone 13, Casper Via X20, General Mobile GM22 Pro, Reeder P13 Blue Max Pro, Huawei P40 Pro and Oppo Reno 6. In the first stage of the application, the criteria were weighted according to the traditional AHP method. The result of this weighting is shown in Figure 2 below.

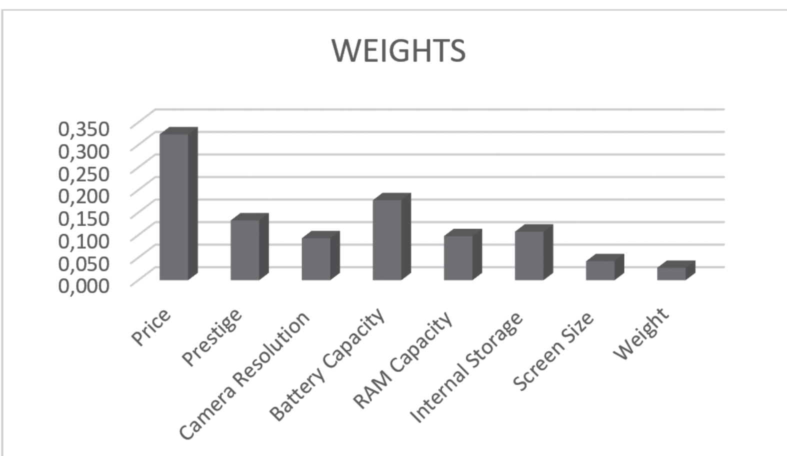

Figure 2: Traditional AHP Criterion Weights

As can be seen from Figure 2, Price criterion is the most important criterion, followed by Battery Capacity and Prestige.

Then, the traditional TOPSIS method was used to rank the alternatives according to the criterias. The result of this ranking is shown in Figure 3 below.

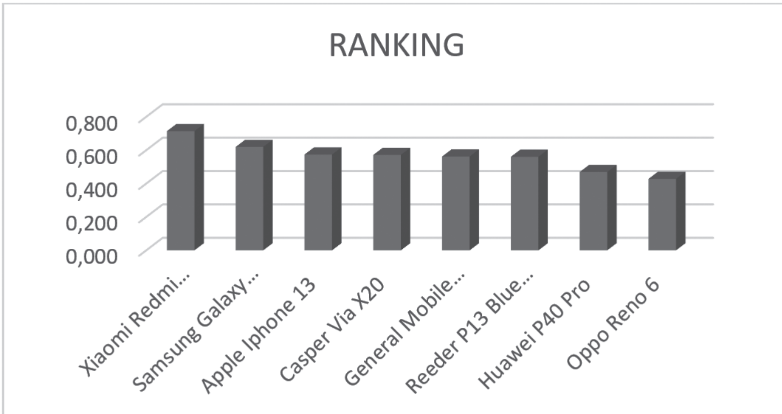

Figure 3: Ranking of Traditional TOPSIS Alternatives

As shown in Figure 3, the Xiaomi brand phone has become the leader in the ranking by outperforming other brands.

In the second stage of the application, the criteria were weighted according to the fuzzy AHP method. The result of this weighting is shown in Figure 4 below.

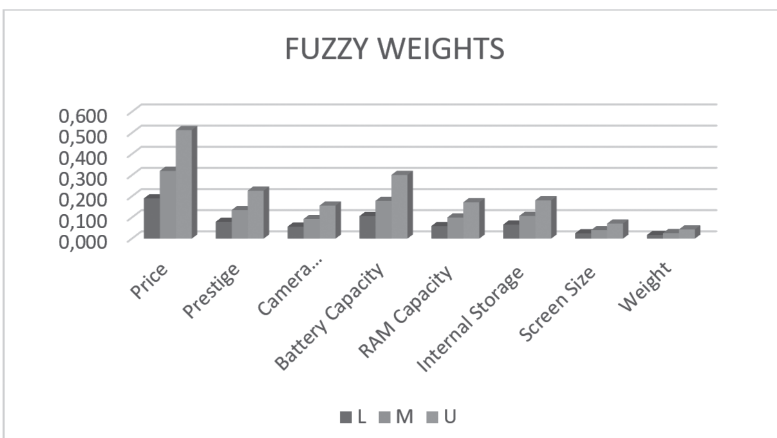

Figure 4: Fuzzy AHP Criterion Weights

As can be seen from Figure 4, Price remains the most important criterion, followed by Battery Capacity and Prestige.

Then, fuzzy TOPSIS method was used to rank the alternatives according to the criteria. The result of this ranking is shown in Figure 5 below.

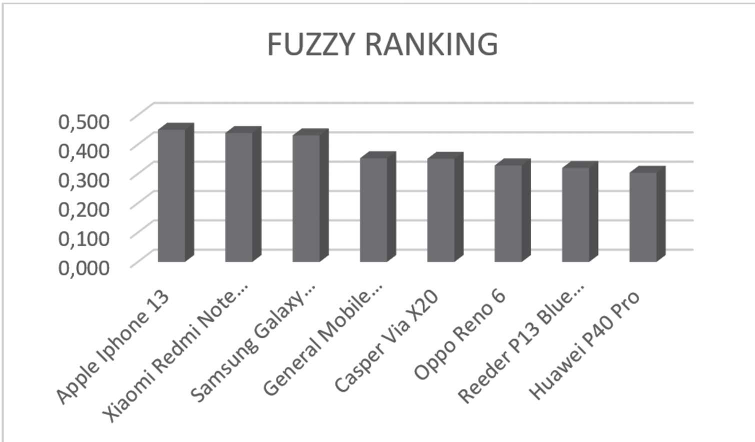

Figure 5: Ranking of Fuzzy TOPSIS Alternatives

As shown in Figure 5, contrary to the traditional method, Apple has been the leader among the alternatives. It can be seen as a confusing issue that the Apple product, which is generally poorly rated for Price and Battery Capacity, is in the first place. However, it is among the possibilities that Apple will come out first, as scoring is made on a price/performance basis. However, it can be said that there is no strong difference between it and its closest competitors due to its Battery Capacity.

According to these results, it is seen that there is an important difference between the traditional method and the fuzzy method. Although the difference causes a serious change on the ranking, it actually consists of binary or triple displacement of results that are numerically close to each other. From this, it is possible to conclude that both methods work similarly, but alternatives with close results are replaced as the cost of uncertainty. It has been mentioned in the studies that fuzzy methods give better results. This study proved that the main reason for this is to take into account the uncertainty. As a further study, the relative ranking of different results can be examined.

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